

DSSSB AE/JE 2026

COMPLETE ENGINEERING MATHEMATICS NOTES

Complete Theory • Detailed Hinglish Explanation • Step-by-Step Derivations
Formula Sheets • Important Derivations • Practice Questions

Prepared By

CIVILGYAN WITH SATISH

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How to use this book: Read Complete Theory, reinforce it with the Hinglish Explanation, review every equation and reaction row, then attempt all twenty questions without viewing the solutions. Questions are original DSSSB-style practice, not claimed verbatim previous-year questions.

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Core Mathematical Constants and Conventions

1. π approximately 3.14159265
2. e approximately 2.71828183
3. $i^2 = -1$
4. angles in calculus are in radians
5. $\log$ means natural logarithm unless stated
6. C denotes an arbitrary constant.

How to Study This Book

Exam Method: Definitions, Diagrams, Conditions and Checks

Begin every mathematics problem by identifying the unknown, domain, given conditions and required result. Draw the curve, region, solid, characteristic path or complex-plane picture whenever geometry is involved. Write the governing definition or theorem before selecting a formula.

Check theorem hypotheses such as continuity, differentiability, exactness, analyticity and boundary conditions. Keep algebra symbolic until the final substitution. For calculus, inspect domain and one-sided behaviour before differentiating or integrating.

For ODEs and PDEs, classify the equation before choosing a method. For multiple integrals, sketch the region and write the differential element with its Jacobian. For complex analysis, choose Cartesian form for addition and polar form for products, powers and roots.

Verify answers using dimensions where applicable, derivative substitution, limiting behaviour, symmetry and sign. In MCQs, eliminate options using definitions and standard results before performing long calculations. Maintain an error log of missed signs, limits, constants, Jacobians and theorem conditions.

Re-solve errors after one day and one week. In the examination, secure direct conceptual questions first, return to calculations in a second pass, and reserve time for option-number and negative-marking checks.

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CHAPTER 1 — CURVE TRACING AND APPLICATIONS OF DEFINITE INTEGRALS

1. Introduction

Curve tracing turns an equation into a reliable geometric picture through domain, symmetry, intercepts, tangents, monotonicity, concavity and asymptotes. Definite integrals then measure area, arc length, surface area and volume. These tools support alignment design, section properties, tanks and graphical engineering models.

2. Complete Theory

Domain, intercepts and symmetry

A function is traced only on its domain. Exclude zero denominators, negative arguments of even roots and invalid logarithms.

Set $y=0$ or $x=0$ for intercepts. Replacing x by $-x$ tests y -axis symmetry; y by $-y$ tests x -axis symmetry; both tests origin symmetry.

Periodic curves repeat after a least positive period.

Hinglish Explanation

Graph se pehle allowed x -values aur anchor intercepts nikalo. Sign replacement mirror test hai.

Symmetry work half kar sakti hai, par domain aur branches phir bhi check karo.

Tangents and singular points

For $F(x,y)=0$, $dy/dx = -F_x/F_y$ when F_y is nonzero. A vertical tangent occurs when $F_y=0$ with F_x nonzero.

If both vanish, the point may be singular and lowest-degree terms give possible tangents. In polar form $r=f(\theta)$, $r=0$ locates the pole.

Hinglish Explanation

Tangent instant direction hai. $0/0$ aaye to derivative force mat karo; point special ho sakta hai aur lowest-degree terms se multiple tangents milte hain.

Monotonicity, extrema and concavity

A differentiable function increases where $f'>0$ and decreases where $f'<0$. A sign change $+$ to $-$ gives a local maximum and $-$ to $+$ a minimum.

Concavity is upward for $f''>0$ and downward for $f''<0$. An inflection point requires concavity change; $f''=0$ alone is insufficient.

Hinglish Explanation

f' uphill-downhill traffic signal hai. f'' bend batata hai.

Stationary point ko maximum bolne se pehle f' sign change aur inflection ke liye f'' sign change dekho.

Asymptotes and branch behaviour

A vertical asymptote $x=a$ requires unbounded one-sided behaviour. Horizontal $y=L$ follows from $\lim f=L$ at infinity.

For oblique $y=mx+c$, $m=\lim f/x$ and $c=\lim(f-mx)$, when finite. Factor and cancel before declaring an asymptote.

Hinglish Explanation

Asymptote woh line hai jiske paas curve approach karta hai. Denominator zero automatic asymptote nahi; cancellation aur one-sided limits zaroor dekho.

Area and polar area

If $f \geq g$ on $[a, b]$, area is $\int_a^b (f-g) dx$. Horizontal strips use right minus left with dy .

Split wherever boundaries cross. Geometric area uses absolute contributions.

In polar coordinates $A = \frac{1}{2} \int r^2 d\theta$.

Hinglish Explanation

Area ka golden rule top-bottom ya right-left hai. Crossing par interval split karo, warna signed cancellation galat geometric area dega.

Arc length, surfaces and volumes

For $y=f(x)$, $ds = \sqrt{1+(y')^2} dx$ and $L = \int ds$. Rotation about x-axis gives $S = 2\pi \int y ds$.

Washer volume is $\pi \int (R^2 - r^2) dx$; shell volume is $2\pi \int (\text{radius})(\text{height}) dx$. Sketch one generating strip before choosing a method.

Hinglish Explanation

Length Pythagoras se aata hai. Volume mein washer slices ya shells choose karo jo boundaries simple rakhe; inner radius kabhi mat bhoolo.

4. Mathematical Formula Section

Formula Sheet

1. $f(-x)=f(x) \rightarrow y\text{-axis symmetry}$
2. $f(-x)=-f(x) \rightarrow \text{origin symmetry}$
3. $dy/dx = -F_x/F_y$
4. $f'(x)=0$ gives a critical candidate
5. $f''>0$ concave upward
6. $m = \lim_{x \rightarrow \infty} f(x)/x$
7. $c = \lim_{x \rightarrow \infty} [f(x) - mx]$
8. $A = \int_a^b f(x) dx$
9. $A_{\text{polar}} = (1/2) \int_{\theta_1}^{\theta_2} r^2 d\theta$
10. $L = \int_a^b \sqrt{1+(y')^2} dx$
11. $S_x = 2\pi \int y ds$
12. $V_{\text{washer}} = \pi \int (R^2 - r^2) dx$
13. $V_{\text{shell}} = 2\pi \int (\text{radius})(\text{height}) dx$

5. Formula Derivations and Solved Examples

Area is the limit of rectangle sums $\sum f(x_i) \Delta x$, giving $\int_a^b f(x) dx$. Thus area under $y=x^2$ on $[0,1]$ is $[x^3/3]_0^1 = 1/3$. For a small arc, $(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2$; divide by Δx squared and pass to the limit to obtain $ds = \sqrt{1+(dy/dx)^2} dx$. For $y=x$ on $[0,1]$, $L = \sqrt{2}$. Rotating $y=f(x)$ gives a disk $dV = \pi y^2 dx$. For $y=x$, $0 \leq x \leq h$, $V = \pi \int_0^h x^2 dx = \pi h^3/3$.

6. Practical Applications

- Road profiles use slope and curvature.
- Reservoir capacity uses integrated cross-sections.
- Arc length estimates cable and pipe lengths.
- Volumes of revolution model tanks and shafts.

7. Short Tricks

- Use symmetry to trace one branch and reflect it.
- Factor before testing asymptotes.
- Choose dx or dy by fewer boundary changes.

- Sketch the rotating strip before choosing washer or shell.

8. Concept Diagram

TRACE IN ORDER

Equation $\rightarrow$ Domain $\rightarrow$ Symmetry $\rightarrow$ Intercepts $\rightarrow$ Tangents

$\rightarrow$ sign of f' $\rightarrow$ sign of f'' $\rightarrow$ Asymptotes $\rightarrow$ Branches

Area: draw one strip and split at every intersection.

9. Comparison Table

Method	Element	Use
Vertical area	$(\text{top-bottom})dx$	x-simple region
Horizontal area	$(\text{right-left})dy$	y-simple region
Washer	$\pi(R^2 - r^2)dx$	strip perpendicular to axis
Shell	$2\pi(\text{radius})(\text{height})dx$	strip parallel to axis

10. Common Mistakes

- Ignoring domain before tracing.
- Calling $f''=0$ an inflection without sign change.
- Using signed integral for geometric area.
- Dropping inner radius in washers.

11. Important DSSSB Exam Points

- Domain and symmetry precede differentiation.
- Extrema need derivative sign analysis.
- Area may require interval splitting.
- Know parabola, ellipse, hyperbola and common polar curves.

12. Chapter Revision Sheet

Trace in the fixed sequence domain, symmetry, intercepts, tangents, derivative signs, concavity and asymptotes. Draw strips before definite-integral geometry. Memorize area, polar area, arc length, surface and volume elements.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. $f(-x)=f(x)$ gives symmetry about

- (A) x-axis
- (B) y-axis
- (C) origin
- (D) $y=x$

Correct Answer: (B)

This is the even-function test.

Q2. Origin on $y=x^3$ is a

- (A) maximum
- (B) minimum
- (C) inflection
- (D) asymptote

Correct Answer: (C)

Concavity changes at zero.

Q3. Vertical asymptote of $1/(x-2)$ is

- (A) $y=2$
- (B) $x=2$
- (C) $x=0$
- (D) $y=0$

Correct Answer: (B)

The function is unbounded near $x=2$.

Q4. Horizontal asymptote of $(2x+1)/(x+3)$ is

- (A) $y=0$
- (B) $x=2$
- (C) $y=2$
- (D) $y=3$

Correct Answer: (C)

Use the ratio of leading coefficients.

Q5. f' changes + to - at a point: it is a

- (A) minimum
- (B) maximum
- (C) saddle
- (D) asymptote

Correct Answer: (B)

The function changes increasing to decreasing.

Q6. Inflection requires change in

- (A) domain
- (B) concavity
- (C) period
- (D) intercept

Correct Answer: (B)

That is the defining condition.

Q7. Area between $y=x$ and $y=x^2$ on $[0,1]$ is

- (A) $1/2$
- (B) $1/3$
- (C) $1/6$
- (D) $5/6$

Correct Answer: (C)

Integral of $x-x^2$ is $1/6$.

Q8. Polar area element is

- (A) $r \, d\theta$
- (B) $r^2 d\theta$
- (C) $\frac{r^2 d\theta}{2}$
- (D) $2\pi r \, d\theta$

Correct Answer: (C)

A thin sector has area one-half r squared $d\theta$.

Q9. Arc element is

- (A) $(1+y')dx$
- (B) $\sqrt{1+y'}dx$
- (C) $\sqrt{1+y'^2}dx$
- (D) $y'dx$

Correct Answer: (C)

It follows from Pythagoras.

Q10. Volume of $y=x$ on $[0,1]$ about x-axis is

- (A) π
- (B) $\pi/2$
- (C) $\pi/3$
- (D) $2\pi/3$

Correct Answer: (C)

Integrate πx squared.

Q11. For $y=x^2$, f'' equals

- (A) 0
- (B) 1
- (C) 2
- (D) $2x$

Correct Answer: (C)

Differentiate twice.

Q12. $y=|x|$ has at origin a

- (A) corner
- (B) maximum
- (C) asymptote
- (D) horizontal tangent

Correct Answer: (A)

One-sided slopes differ.

Q13. $xy=1$ is symmetric about

- (A) x-axis
- (B) y-axis
- (C) origin
- (D) none

Correct Answer: (C)

Both sign changes preserve xy .

Q14. Oblique asymptote of $x+1/x$ is

- (A) $y=0$
- (B) $x=0$
- (C) $y=x$
- (D) $y=x+1$

Correct Answer: (C)

The difference from x tends to zero.

Q15. Geometric area uses

- (A) integral f only
- (B) integral $|f|$
- (C) endpoint product
- (D) zero

Correct Answer: (B)

All pieces contribute positively.

Q16. Surface about x -axis contains factor

- (A) πy
- (B) $2\pi y$
- (C) πy^2
- (D) $2\pi x$

Correct Answer: (B)

Circumference is $2\pi y$.

Q17. Shell radius about y -axis is

- (A) y
- (B) x
- (C) y'
- (D) $1/x$

Correct Answer: (B)

It is distance from the y -axis.

Q18. Period of $\sin x$ is

- (A) $\pi/2$
- (B) π
- (C) 2π
- (D) 4π

Correct Answer: (C)

$\sin$ repeats after 2π .

Q19. At zero, x^4 has a

- (A) maximum
- (B) minimum
- (C) asymptote
- (D) corner

Correct Answer: (B)

x^4 is nonnegative.

Q20. Before area integration determine

- (A) intersections
- (B) only period
- (C) only derivative
- (D) only asymptote

Correct Answer: (A)

Intersections determine limits and ordering.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.

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CHAPTER 2 — ONE VARIABLE CALCULUS AND PARTIAL DIFFERENTIATION

1. Introduction

Calculus formalizes limits, smooth change, approximation and optimization. Partial differentiation extends sensitivity to multivariable engineering models.

2. Complete Theory

Limits and continuity

A limit is the approached value near a point. Continuity at a requires $\lim_{x \rightarrow a} f(x) = f(a)$.

Standard limits and valid algebra resolve indeterminate forms; L'Hopital applies only to forms such as $0/0$ or infinity/infinity under its hypotheses.

Hinglish Explanation

Limit point ke paas behaviour hai. $0/0$ answer nahi, simplification ka signal hai.

Continuity mein left limit, right limit aur actual value same honi chahiye.

Differentiability and rules

Differentiability requires one finite common slope and implies continuity, not conversely. Product, quotient and chain rules handle combinations.

Implicit differentiation treats y as $y(x)$; logarithmic differentiation handles variable powers.

Hinglish Explanation

Smooth single slope differentiability hai. Chain rule mein inner derivative aur implicit equation mein dy/dx factor mat bhoolo.

Higher derivatives and mean-value theorems

Higher derivatives measure changing slope. Rolle and mean-value theorems require continuity on $[a,b]$ and differentiability on (a,b) ; MVT gives an interior tangent parallel to the endpoint secant.

Leibniz theorem differentiates products n times.

Hinglish Explanation

Theorem formula se pehle conditions check karo. MVT ka visual: beech mein ek tangent secant ke parallel zaroor milta hai.

Taylor and Maclaurin series

Taylor expansion about a uses derivatives at a ; Maclaurin uses $a=0$. Standard series for exponential, sine, cosine, logarithm and binomial functions approximate locally with a remainder controlling error.

Hinglish Explanation

Series function ka local polynomial model hai. Signs, factorial aur convergence range sabse common traps hain.

Partial derivatives, total differential and Euler theorem

For $z=f(x,y)$, f_x varies x while y is fixed. $dz=f_{xdx}+f_{ydy}$ is the first-order combined change.

If $f(tx,ty)=t^n f(x,y)$, Euler theorem gives $xf_x+yf_y=nf$.

Hinglish Explanation

Partial derivative mein ek knob chalao, doosra freeze. Total differential dono small effects add karta hai.

Euler scaling ko derivatives se connect karta hai.

Multivariable extrema

At an interior stationary point $f_x = f_y = 0$. $D = f_{xx}f_{yy} - f_{xy}^2$ classifies: $D > 0$ with $f_{xx} > 0$ minimum, $D > 0$ with $f_{xx} < 0$ maximum, $D < 0$ saddle.

Constraints use $\text{grad } f = \lambda \text{ grad } g$.

Hinglish Explanation

Surface par stationary point bowl, hill ya saddle ho sakta hai. Hessian type batata hai; constraint fence ho to Lagrange multiplier use karo.

4. Mathematical Formula Section

Formula Sheet

1. $\lim_{x \rightarrow 0} \sin x/x = 1$
2. $\lim_{x \rightarrow 0} (e^x - 1)/x = 1$
3. $(uv)' = u'v + uv'$
4. $(u/v)' = (vu' - uv')/v^2$
5. $dy/dx = -F_x/F_y$
6. $e^x = \sum x^n/n!$
7. $\sin x = x - x^3/3! + x^5/5! - \dots$
8. $\cos x = 1 - x^2/2! + x^4/4! - \dots$
9. $dz = f_{xdx} + f_{ydy}$
10. $xf_x + yf_y = nf$
11. $D = f_{xx}f_{yy} - f_{xy}^2$
12. $\text{grad } f = \lambda \text{ grad } g$

5. Formula Derivations and Solved Examples

Taylor gives $f(x) = f(a) + (x-a)f'(a) + (x-a)^2 f''(a)/2! + \dots$. With $f = e^x, a=0$ this becomes $1 + x + x^2/2! + \dots$. For $f = x^2y + y^3$, $f_x = 2xy$ and $f_y = x^2 + 3y^2$, hence $dz = 2xy dx + (x^2 + 3y^2)dy$. It is degree 3 and $xf_x + yf_y = 3f$. For minimizing $x^2 + y^2$ subject to $x + y = 1$, Lagrange equations give $x = y = 1/2$ and minimum $1/2$.

6. Practical Applications

- Error propagation uses total differentials.
- Optimization reduces cost and weight.
- Series linearize nonlinear models.

- Gradients describe field direction.

7. Short Tricks

- Normalize to standard limits.
- Use total degree to spot homogeneity.
- Memorize four core Maclaurin series.
- Solve constraint and gradient equations together.

8. Concept Diagram

LIMIT $\rightarrow$ CONTINUITY $\rightarrow$ DIFFERENTIABILITY $\rightarrow$ LOCAL LINEAR MODEL

$\rightarrow$ Taylor higher-order model

Two variables: partial slopes $\rightarrow$ gradient $\rightarrow$ Hessian classification

9. Comparison Table

Idea	Condition	Result
Continuity	limit=value	no break
Differentiability	common finite slope	implies continuity
Partial change	one variable varies	f_x or f_y
Total change	all vary	dz =sum effects

10. Common Mistakes

- Using L'Hopital without an indeterminate form.
- Assuming continuity implies differentiability.
- Dropping chain factors.
- Treating $D=0$ as conclusive.

11. Important DSSSB Exam Points

- Standard limits and series are high-yield.
- Differentiability implies continuity only one way.
- Euler theorem and Hessian are frequent.
- Total differential gives first-order error.

12. Chapter Revision Sheet

Limits define continuity and derivatives. Use chain and implicit rules carefully. Taylor approximates functions. Partial derivatives, Euler theorem, Hessian and Lagrange multipliers form the multivariable core.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. $\lim_{x \rightarrow 0} \sin 3x/x$

- (A) 0
- (B) 1
- (C) 3
- (D) infinity

Correct Answer: (C)

Normalize by $3x$.

Q2. Differentiability implies

- (A) continuity
- (B) maximum
- (C) periodicity
- (D) discontinuity

Correct Answer: (A)

It is a necessary consequence.

Q3. Derivative e^{2x}

- (A) e^{2x}
- (B) $2e^{2x}$
- (C) xe^{2x}
- (D) $2x$

Correct Answer: (B)

Chain rule.

Q4. For $x^2 + y^2 = 1$, y'

- (A) x/y
- (B) $-x/y$
- (C) $-y/x$
- (D) 1

Correct Answer: (B)

Implicit differentiation.

Q5. Derivative $\ln x$

- (A) x
- (B) $1/x$
- (C) $\ln x$
- (D) e^x

Correct Answer: (B)

Standard result.

Q6. $\lim (e^x - 1)/x$

- (A) 0
- (B) 1
- (C) e
- (D) infinity

Correct Answer: (B)

Standard limit.

Q7. Maclaurin centre

- (A) 0
- (B) 1
- (C) -1
- (D) infinity

Correct Answer: (A)

Maclaurin is Taylor at zero.

Q8. x^2 coefficient in e^x

- (A) 1
- (B) $1/2$
- (C) $1/3$
- (D) 2

Correct Answer: (B)

It is $1/2$ factorial.

Q9. $\partial_x x^2y$

- (A) x^2
- (B) $2xy$
- (C) x^2y
- (D) $2x$

Correct Answer: (B)

Hold y fixed.

Q10. Euler theorem

- (A) $f_x + f_y = n$
- (B) $xf_x + yf_y = nf$
- (C) $xyf_{xy} = f$
- (D) $f_{x^2y} = f$

Correct Answer: (B)

Homogeneous degree n identity.

Q11. For x^2+y^2 , D

- (A) -4
- (B) 0
- (C) 4
- (D) 8

Correct Answer: (C)

2 times 2.

Q12. $D < 0$ indicates

- (A) maximum
- (B) minimum
- (C) saddle
- (D) constant

Correct Answer: (C)

Curvatures oppose.

Q13. $d(xy)$

- (A) $x dx + y dy$
- (B) $y dx + x dy$
- (C) $xy dx dy$
- (D) $dx dy$

Correct Answer: (B)

Use partials.

Q14. Rolle endpoints are

- (A) equal
- (B) opposite
- (C) zero only
- (D) undefined

Correct Answer: (A)

$f(a)=f(b)$.

Q15. MVT slope

- (A) 0
- (B) secant slope
- (C) $f(a)$
- (D) $f(b)$

Correct Answer: (B)

Interior tangent equals secant.

Q16. Derivative x^x

- (A) $x^x(1+\ln x)$
- (B) $x^x \ln x$
- (C) $x x^{x-1}$
- (D) 1

Correct Answer: (A)

Log differentiation.

Q17. Mixed partial equality needs

- (A) regularity
- (B) periodicity
- (C) homogeneity
- (D) linearity

Correct Answer: (A)

Continuous partials suffice locally.

Q18. Gradient is normal to

- (A) level curve
- (B) axis always
- (C) tangent
- (D) minimum only

Correct Answer: (A)

Level directional change is zero.

Q19. Lagrange condition

- (A) $f=g$
- (B) $\text{grad } f = \lambda \text{ grad } g$
- (C) $\lambda = 0$
- (D) $f_x = 0$ only

Correct Answer: (B)

Gradients are parallel.

Q20. Linear approximation

- (A) $f(a) + f'(a)(x-a)$
- (B) $f'(a)$
- (C) $f(a)x$
- (D) $f''(a)$

Correct Answer: (A)

Tangent-line model.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.

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CHAPTER 3 — ORDINARY DIFFERENTIAL EQUATIONS

1. Introduction

ODEs relate a one-variable function to its derivatives and model growth, cooling, circuits and vibration.

2. Complete Theory

Order, degree and solutions

Order is the highest derivative. Degree is its power after polynomializing derivatives.

A general solution contains arbitrary constants; data select a particular solution.

Hinglish Explanation

Order highest derivative hai. Degree radical/fraction remove karne ke baad define hoti hai.

Initial data constants fix karti hai.

Separable and homogeneous equations

$dy/dx=g(x)h(y)$ separates. For $dy/dx=F(y/x)$, set $y=vx$ so $dy/dx=v+x dv/dx$.

Hinglish Explanation

x aur y sides separate karo. y/x form mein $y=vx$ key substitution hai; v term mat bhoolo.

Linear, Bernoulli and exact equations

$y'+Py=Q$ uses IF= $e^{\int P dx}$. Bernoulli uses $v=y^{1-n}$.

$Mdx+Ndy=0$ is exact if $M_y=N_x$ and integrates to a potential.

Hinglish Explanation

IF left side ko product derivative banata hai. Exact equation total differential hoti hai.

Constant-coefficient higher order

For $F(D)y=X$, solution is CF+PI. Auxiliary roots create exponential, repeated-root or damped sine-cosine CF forms.

Hinglish Explanation

CF natural response, PI forcing response. Roots ka type solution ka shape batata hai.

Particular integral and resonance

For e^{ax} , $PI=e^{ax}/F(a)$ when $F(a)$ nonzero. If zero, resonance needs multiplication by x according to multiplicity.

Hinglish Explanation

Denominator zero infinity nahi; resonance hai. Matching CF mode ke liye x correction lagta hai.

Engineering models

Growth $y'=ky$ is exponential. Newton cooling uses $T'=-k(T-T_s)$.

Undamped vibration is $y''+\omega^2 y=0$.

Hinglish Explanation

Equation change law hai, initial condition starting state. Dono se actual prediction milti hai.

4. Mathematical Formula Section

Formula Sheet
1. $y' + Py = Q$
2. $IF = e^{\int P dx}$
3. $y(IF) = \int Q(IF) dx + C$
4. $v = y^{1-n}$
5. exact if $M_y = N_x$
6. $F(D)y = X$
7. repeated root $a \rightarrow e^{ax}(C_1 + C_2x + \dots)$
8. $\alpha + i\beta \rightarrow e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$
9. $y = y_0 e^{kt}$
10. $T - T_s = Ce^{-kt}$

5. Formula Derivations and Solved Examples

Multiplying $y' + Py = Q$ by μ gives $d(\mu y)/dx$ if $\mu' = P\mu$, hence $\mu = e^{\int P dx}$. Example $y' + y = e^x$ gives $y = e^{x/2} + Ce^{-x}$. For $y'' - 3y' + 2y = e^x$, roots 1, 2 give CF $= C_1 e^x + C_2 e^{2x}$; resonance with e^x gives PI $= -xe^x$.

6. Practical Applications

- Cooling transients.
- Spring vibration.
- RL/RLC circuits.
- Growth and decay.

7. Short Tricks

- Classify before solving.
- Write root multiplicities.
- Count resonance multiplicity.
- Apply data after general solution.

8. Concept Diagram

CLASSIFY FIRST

First order $\rightarrow$ separable / homogeneous / linear-Bernoulli / exact

Higher order $\rightarrow$ auxiliary roots $\rightarrow$ CF $\rightarrow$ forcing $\rightarrow$ PI $\rightarrow$ conditions

9. Comparison Table

Type	Recognition	Tool
Separable	$g(x)h(y)$	separate
Linear	$y' + Py = Q$	IF
Bernoulli	$y' + Py = Qy^n$	power substitution
Exact	$M_y = N_x$	potential

10. Common Mistakes

- Defining degree too early.
- Forgetting v in $y=vx$ derivative.
- Omitting integration constants.
- Using direct PI at resonance.

11. Important DSSSB Exam Points

- IF and exactness are central.
- CF depends on roots.
- PI depends on forcing.
- Order predicts constant count.

12. Chapter Revision Sheet

Classify first-order equations before integrating. For constant coefficients obtain CF, then PI and finally apply conditions.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. Order $y''' + y = 0$

- (A) 1
- (B) 2
- (C) 3
- (D) undefined

Correct Answer: (C)

Highest derivative third.

Q2. Degree $(y'')^3 + y' = 0$

- (A) 1
- (B) 2
- (C) 3
- (D) undefined

Correct Answer: (C)

Highest derivative power 3.

Q3. $y'=0$ gives

- (A) $y=x$
- (B) $y=C$
- (C) e^x
- (D) 0 only

Correct Answer: (B)

Zero slope means constant.

Q4. $y'=xy$ is

- (A) separable
- (B) second order
- (C) exact only
- (D) linear in x

Correct Answer: (A)

$dy/y=x \, dx$.

Q5. For $F(y/x)$ use

- (A) $y=vx$
- (B) $y=v+x$
- (C) $y=v/x$
- (D) $x=v+y$

Correct Answer: (A)

Standard homogeneous substitution.

Q6. IF of $y'+2y=x$

- (A) e^x
- (B) e^{2x}
- (C) x^2
- (D) $2e^x$

Correct Answer: (B)

Exponentiate integral $2dx$.

Q7. Bernoulli substitution

- (A) y^n
- (B) y^{1-n}
- (C) xy
- (D) $\ln x$

Correct Answer: (B)

It makes the equation linear.

Q8. Exactness

- (A) $M_x=N_y$
- (B) $M_y=N_x$
- (C) $M=N$
- (D) $MN=1$

Correct Answer: (B)

Cross partial test.

Q9. $y'+y=0$

- (A) Ce^x
- (B) Ce^{-x}
- (C) Cx
- (D) C/x

Correct Answer: (B)

Separate or use IF.

Q10. Auxiliary equation $y''+y=0$

- (A) $m+1$
- (B) m^2+1
- (C) m^2-1
- (D) m

Correct Answer: (B)

Replace D by m.

Q11. Repeated root 2 CF

- (A) $C_1e^{2x}+C_2e^{-2x}$
- (B) $(C_1+C_2x)e^{2x}$
- (C) $\cos 2x$
- (D) C_1+C_2x

Correct Answer: (B)

Repeated root adds x.

Q12. Roots $\pm 3i$ CF

- (A) exponentials
- (B) $C_1\cos 3x + C_2\sin 3x$
- (C) polynomial
- (D) zero

Correct Answer: (B)

Imaginary roots give trig.

Q13. Complete solution

- (A) CF
- (B) PI
- (C) CF+PI
- (D) CF times PI

Correct Answer: (C)

Superposition.

Q14. Cooling rate proportional to

- (A) temperature difference
- (B) time
- (C) mass only
- (D) temperature squared

Correct Answer: (A)

Newton law.

Q15. $y'=ky$ solution

- (A) $y_0 + kt$
- (B) $y_0 e^{kt}$
- (C) kt^2
- (D) y_0/k

Correct Answer: (B)

Exponential growth.

Q16. Roots $y''-y=0$

- (A) 0,1
- (B) 1,-1
- (C) i,-i
- (D) 1,1

Correct Answer: (B)

Factor $m^2 - 1$.

Q17. Third-order constants

- (A) 1
- (B) 2
- (C) 3
- (D) 0

Correct Answer: (C)

One per order.

Q18. Initial data determine

- (A) order
- (B) constants
- (C) exactness
- (D) variable

Correct Answer: (B)

They select a particular solution.

Q19. PI for e^{ax} , $F(a) \neq 0$

- (A) $F(a)e^{ax}$
- (B) $e^{ax}/F(a)$
- (C) xe^{ax}
- (D) 0

Correct Answer: (B)

Operator rule.

Q20. Resonance when

- (A) $F(a)=0$
- (B) forcing zero
- (C) roots distinct
- (D) first order

Correct Answer: (A)

Forcing matches a natural mode.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.

CHAPTER 4 — PARTIAL DIFFERENTIAL EQUATIONS

1. Introduction

PDEs describe fields depending on several variables, including temperature, waves and potential.

2. Complete Theory

Definitions and formation

For $z(x,y)$, $p=z_x$ and $q=z_y$. Order and degree follow highest partial derivatives.

Differentiate a family and eliminate arbitrary constants or functions.

Hinglish Explanation

PDE surface ka relation hai; p, q two coordinate slopes. Arbitrary object eliminate karne tak differentiate karo.

Lagrange first-order equation

$Pp+Qq=R$ uses $dx/P=dy/Q=dz/R$. Two independent integrals $u=a, v=b$ combine as $\phi(u,v)=0$.

Multipliers help form integrable combinations.

Hinglish Explanation

Subsidiary ratios characteristic paths hain. Do independent invariants compulsory hain.

Nonlinear first order

Clairaut type $z=px+qy+f(p,q)$ has complete integral $z=ax+by+f(a,b)$. General nonlinear equations may use Charpit characteristics.

Hinglish Explanation

Standard form pehchano; p, q ko constants a, b rakhkar complete integral milta hai.

Second-order classification

For $Au_{xx}+2Bu_{xy}+Cu_{yy}+\dots=0$, B^2-AC negative, zero or positive gives elliptic, parabolic or hyperbolic type.

Hinglish Explanation

Mixed term ka coefficient $2B$ hai. Pehle A, B, C correctly identify karo.

Physical PDEs

Laplace $u_{xx}+u_{yy}=0$ is steady elliptic; heat $u_t=k u_{xx}$ is diffusive parabolic; wave $u_{tt}=c^2 u_{xx}$ is propagating hyperbolic.

Hinglish Explanation

Laplace steady, heat smoothing, wave propagation - yeh memory map rakho.

Separation and characteristics

Assume $u=XT$, separate into ODEs and apply boundary data to determine eigenvalues. Characteristics carry data for first-order and hyperbolic equations.

Hinglish Explanation

Product assumption PDE ko ODEs mein todta hai; boundary conditions allowed modes select karti hain.

4. Mathematical Formula Section

Formula Sheet
1. $p = z_x$
2. $q = z_y$
3. $Pp + Qq = R$
4. $dx/P = dy/Q = dz/R$
5. $Au_{xx} + 2Bu_{xy} + Cu_{yy} = 0$
6. elliptic $B^2 - AC < 0$
7. parabolic $= 0$
8. hyperbolic > 0
9. Laplace $u_{xx} + u_{yy} = 0$
10. heat $u_t = k u_{xx}$
11. wave $u_{tt} = c^2 u_{xx}$
12. $u = XT$

5. Formula Derivations and Solved Examples

For heat equation put $u = XT$: $T'/(kT) = X''/X = -\lambda$. Thus $X'' + \lambda X = 0$ and $T' + k\lambda T = 0$. Zero-end boundary data give $\lambda_n = (n\pi/L)^2$, $X_n = \sin(n\pi x/L)$, $T_n = e^{-k(n\pi/L)^2 t}$. For $xp + yq = z$, subsidiary equations yield $y/x = C_1$, $z/x = C_2$, hence $z = x \phi(y/x)$.

6. Practical Applications

- Steady temperature.
- Diffusion.
- Waves and acoustics.
- Pollutant transport.

7. Short Tricks

- Write A, 2B, C first.
- Try pairwise subsidiary ratios then multipliers.
- Check invariant independence.
- Use steady-diffusion-wave map.

8. Concept Diagram

$B^2 - AC < 0$ elliptic $\rightarrow$ equilibrium

$B^2 - AC = 0$ parabolic $\rightarrow$ diffusion

$B^2 - AC > 0$ hyperbolic $\rightarrow$ waves

9. Comparison Table

Equation	Type	Meaning
Laplace	elliptic	steady
Heat	parabolic	diffusion
Wave	hyperbolic	propagation
Lagrange	first order	characteristics

10. Common Mistakes

- Using full mixed coefficient as B.
- Finding only one invariant.
- Ignoring boundary data.
- Confusing order and derivative count.

11. Important DSSSB Exam Points

- Classification is direct.
- Lagrange ratios are essential.
- Know standard physical PDEs.
- Separation gives eigenvalue ODEs.

12. Chapter Revision Sheet

Form PDEs by elimination. Solve Lagrange equations through two invariants. Classify by $B^2 - AC$ and recognize Laplace, heat and wave equations.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. p denotes

- (A) z
- (B) z_x
- (C) z_y
- (D) z_{xy}

Correct Answer: (B)

Definition.

Q2. Lagrange form

- (A) $Pp + Qq = R$
- (B) $pq = 0$
- (C) $u_{xx} = 0$
- (D) $z = C$

Correct Answer: (A)

Standard form.

Q3. Subsidiary system

- (A) $Pdx = Qdy$
- (B) $dx/P = dy/Q = dz/R$
- (C) $dx + dy = dz$
- (D) $P/Q = R$

Correct Answer: (B)

Characteristic ratios.

Q4. Need invariants

- (A) 1
- (B) 2
- (C) 3
- (D) 0

Correct Answer: (B)

Two independent integrals.

Q5. Elliptic discriminant

- (A) positive
- (B) zero
- (C) negative
- (D) one

Correct Answer: (C)

Definition.

Q6. Heat type

- (A) elliptic
- (B) parabolic
- (C) hyperbolic
- (D) ODE

Correct Answer: (B)

Diffusion type.

Q7. Wave type

- (A) elliptic
- (B) parabolic
- (C) hyperbolic
- (D) algebraic

Correct Answer: (C)

Propagation type.

Q8. Laplace models

- (A) steady potential
- (B) growth
- (C) translation
- (D) interpolation

Correct Answer: (A)

Source-free steady field.

Q9. Separation assumes

- (A) $X+T$
- (B) XT
- (C) X/T only
- (D) constant

Correct Answer: (B)

Product form.

Q10. Boundary data determine

- (A) eigenvalues
- (B) order
- (C) variables
- (D) notation

Correct Answer: (A)

They select modes.

Q11. k in heat equation

- (A) diffusivity
- (B) frequency
- (C) pressure
- (D) length

Correct Answer: (A)

Physical coefficient.

Q12. Order $u_{xxy} + u = 0$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C)

Three differentiations.

Q13. Degree $(u_{xx})^2 + u_y = 0$

- (A) 1
- (B) 2
- (C) 3
- (D) undefined

Correct Answer: (B)

Highest-order derivative squared.

Q14. $u_{xx} + u_{yy} = 0$

- (A) heat
- (B) wave
- (C) Laplace
- (D) transport

Correct Answer: (C)

Standard Laplace.

Q15. Real distinct characteristics

- (A) elliptic
- (B) parabolic
- (C) hyperbolic
- (D) none

Correct Answer: (C)

Hyperbolic type.

Q16. Clairaut uses p, q as

- (A) constants
- (B) coordinates
- (C) zero
- (D) functions only

Correct Answer: (A)

Complete integral parameters.

Q17. Characteristics carry

- (A) solution data
- (B) units
- (C) randomness
- (D) page numbers

Correct Answer: (A)

Information paths.

Q18. Boundary problem needs

- (A) PDE plus data
- (B) PDE name
- (C) no domain
- (D) one derivative

Correct Answer: (A)

Data determine solution.

Q19. Separation yields

- (A) ODEs
- (B) no equations
- (C) matrix only
- (D) inequality

Correct Answer: (A)

Variables separate.

Q20. Parabolic discriminant

- (A) <0
- (B) $=0$
- (C) >0
- (D) undefined

Correct Answer: (B)

Definition.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.

CHAPTER 5 — SOLID GEOMETRY AND MULTIPLE INTEGRALS

1. Introduction

3D geometry describes lines, planes and quadratic surfaces; multiple integrals measure distributed area, volume, mass and centroid.

2. Complete Theory

Vectors, lines and planes

Direction cosines satisfy $l^2 + m^2 + n^2 = 1$. A line is $r = a + tb$.

A plane through r_0 with normal n is $n \cdot (r - r_0) = 0$. Dot products give angles; cross products give normals and areas.

Hinglish Explanation

Line point plus direction hai; plane normal se fixed hota hai. Dot angle aur cross perpendicular/area ke liye use karo.

Distance and intersections

Point-plane distance is absolute substitution divided by normal magnitude. Lines can intersect, be parallel or skew.

Plane angles equal normal angles.

Hinglish Explanation

3D lines skew ho sakti hain. Distance formula mein absolute value aur normal length dono zaroori hain.

Sphere, cone and cylinder

A sphere has fixed centre and radius. A cone is a homogeneous quadratic surface through a vertex.

A cylinder repeats a cross-section along parallel generators.

Hinglish Explanation

Sphere centre-radius, cone vertex rays, cylinder repeated cross-section - shape ko equation se connect karo.

Double integrals

Double integrals are limits of area sums and become iterated integrals. Draw the region; inner limits follow the moving strip.

Fubini permits order change under suitable conditions.

Hinglish Explanation

Tiny tiles ka sum samjho. Limits ratne ke bajay region aur strip draw karo.

Coordinate changes and Jacobian

Polar coordinates give $dA = r dr d\theta$. Generally $dx dy = |J| du dv$.

The Jacobian is local area scaling and cannot be omitted.

Hinglish Explanation

Coordinate change tile size distort karta hai; Jacobian wahi scale factor hai.

Triple integrals and centroid

Cylindrical $dV = r \, dr \, d\theta \, dz$; spherical $dV = r^2 \sin \phi \, dr \, d\phi \, d\theta$. Mass integrates density; centroid equals first moment divided by mass.

Hinglish Explanation

Symmetry pehle use karo. Mass denominator hai aur coordinate-weighted integral first moment.

4. Mathematical Formula Section

Formula Sheet
1. $l^2 + m^2 + n^2 = 1$
2. $r = a + tb$
3. plane $n \cdot (r - r_0) = 0$
4. distance $= ax_0 + by_0 + cz_0 + d / \sqrt{a^2 + b^2 + c^2}$
5. sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$
6. $dA = r \, dr \, d\theta$
7. $dx dy = J \, du \, dv$
8. $dV_{\text{cyl}} = r \, dr \, d\theta \, dz$
9. $dV_{\text{sph}} = r^2 \sin \phi \, dr \, d\phi \, d\theta$
10. $x_{\text{bar}} = (1/M) \int_{\text{volume}} x \, \text{density} \, dV$

5. Formula Derivations and Solved Examples

Polar Jacobian determinant is r , so circle area is $\int_0^{2\pi} \int_0^a r \, dr \, d\theta = \pi a^2$. Spherical integration gives volume $4\pi a^3/3$. For triangle $x \geq 0, y \geq 0, x+y \leq 1$, area is $\int_0^1 \int_0^{1-x} dy \, dx = 1/2$ and centroid is $(1/3, 1/3)$.

6. Practical Applications

- Survey alignment.
- Tank volume.
- Mass properties.
- Axisymmetric flow.

7. Short Tricks

- Use normals for plane angles.
- Exploit symmetry.
- Match coordinates to boundaries.
- Check area/volume dimensions.

8. Concept Diagram

Cartesian $\rightarrow$ box limits

Cylindrical $\rightarrow$ axial symmetry, factor r

Spherical $\rightarrow$ radial symmetry, factor $\rho^2 \sin \phi$

9. Comparison Table

Integral	Domain	Element
Single	interval	dx
Double	region	dA
Triple	solid	dV
Polar	radial region	$r \, dr \, d\theta$

10. Common Mistakes

- Not normalizing direction ratios.
- Dropping absolute Jacobian.
- Skipping region sketch.
- Forgetting coordinate factors.

11. Important DSSSB Exam Points

- Know line-plane-sphere forms.
- Jacobians are compulsory.
- Area, mass and centroid are frequent.

12. Chapter Revision Sheet

Represent lines parametrically and planes by normals. Draw regions. Polar/cylindrical add r ; spherical adds $\rho^2 \sin \phi$.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. Direction cosines

- (A) $\text{sum}=1$
- (B) $\text{squares sum}=1$
- (C) $\text{product}=1$
- (D) equal

Correct Answer: (B)

Unit direction.

Q2. Plane normal

- (A) (x,y,z)
- (B) (a,b,c)
- (C) (d,d,d)
- (D) ones

Correct Answer: (B)

Coefficients.

Q3. Plane angle uses

- (A) points
- (B) normals
- (C) areas
- (D) intercepts

Correct Answer: (B)

Orientation by normal.

Q4. Origin-plane distance

- (A) d
- (B) $|d|/\sqrt{a^2+b^2+c^2}$
- (C) $a+b+c$
- (D) 0

Correct Answer: (B)

Distance formula.

Q5. 3D line

- (A) point+direction
- (B) normal only
- (C) radius
- (D) one scalar

Correct Answer: (A)

Parametric form.

Q6. Noncoplanar lines

- (A) parallel
- (B) skew
- (C) coincident
- (D) normal

Correct Answer: (B)

Definition.

Q7. Sphere $x^2+y^2+z^2=R^2$ centre

- (A) (R,R,R)
- (B) origin
- (C) none
- (D) ones

Correct Answer: (B)

Standard form.

Q8. Polar dA

- (A) $drd\theta$
- (B) $r drd\theta$
- (C) $r^2 drd\theta$
- (D) $\sin \theta$

Correct Answer: (B)

Jacobian r .

Q9. Circle area

- (A) $2\pi a$
- (B) πa^2
- (C) $4\pi a^2$
- (D) a^3

Correct Answer: (B)

Polar integral.

Q10. Cylindrical dV factor

- (A) 1
- (B) r
- (C) r^2
- (D) $\sin \theta$

Correct Answer: (B)

Jacobian r .

Q11. Spherical factor

- (A) ρ
- (B) $\rho^2 \sin \phi$
- (C) $r \sin \theta$
- (D) 1

Correct Answer: (B)

Jacobian.

Q12. Double integral 1

- (A) perimeter
- (B) area
- (C) centroid
- (D) slope

Correct Answer: (B)

Area sum.

Q13. Triple integral 1

- (A) surface
- (B) volume
- (C) angle
- (D) mass always

Correct Answer: (B)

Volume sum.

Q14. Mass

- (A) integral density dV
- (B) density/ V
- (C) volume only
- (D) centroid

Correct Answer: (A)

Definition.

Q15. x centroid

- (A) first moment/M
- (B) M/moment
- (C) max x
- (D) volume

Correct Answer: (A)

Weighted mean.

Q16. Jacobian is

- (A) scale factor
- (B) temperature
- (C) angle only
- (D) mass

Correct Answer: (A)

Local scaling.

Q17. Fubini permits

- (A) order change
- (B) drop limits
- (C) remove density
- (D) dimension change

Correct Answer: (A)

Under conditions.

Q18. Unit right triangle area

- (A) 1
- (B) $1/2$
- (C) 2
- (D) π

Correct Answer: (B)

Base-height/2.

Q19. Plane through origin

- (A) $d=0$
- (B) $a=b=c=0$
- (C) no normal
- (D) $R=0$

Correct Answer: (A)

Substitution.

Q20. Cross magnitude gives

- (A) parallelogram area
- (B) sphere
- (C) line
- (D) point

Correct Answer: (A)

Vector result.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.

CHAPTER 6 — COMPLEX ANALYSIS

1. Introduction

Complex analysis combines algebra, geometry and differentiation for $z=x+iy$ and supports phasors, potential flow and transformations.

2. Complete Theory

Complex numbers and polar form

$z=x+iy$ has conjugate $x-iy$, modulus $\sqrt{x^2+y^2}$ and argument modulo 2π . Polar form $z=re^{i\theta}$ turns products into modulus multiplication and angle addition.

Hinglish Explanation

Argand plane mein z point/vector hai. Conjugate x-axis reflection, modulus length aur argument direction hai; quadrant check karo.

De Moivre theorem and roots

$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$. The n roots have modulus $r^{1/n}$ and arguments $(\theta + 2k\pi)/n$, so they are equally spaced.

Hinglish Explanation

Power mein angle multiply, roots mein divide aur $k=0$ se $n-1$ tak sab roots lo.

Analytic functions and CR equations

For $f=u+iv$, analyticity requires direction-independent derivative. With continuous first partials, $u_x=v_y$ and $u_y=-v_x$ are sufficient locally; $f'=u_x+i v_x$.

Hinglish Explanation

Complex derivative all directions se same hona chahiye. CR equations real-imaginary parts ko lock karti hain.

Harmonic functions

Real and imaginary parts of analytic functions satisfy $u_{xx}+u_{yy}=0$. A harmonic conjugate follows from $v_y=u_x$ and $v_x=-u_y$, up to a constant.

Hinglish Explanation

Laplacian zero harmonic hai. Ek CR equation integrate karo aur doosri se missing function find karo.

Elementary mappings

$w=z+a$ translates, $w=az$ scales/rotates, $w=1/z$ inverts modulus, and Mobius maps generalized circles to generalized circles.

Hinglish Explanation

Mapping plane ko transform karta hai. Boundary points aur modulus-angle effect se image region samjho.

Contours and residues

Analytic functions have zero integral around closed contours in simply connected domains. Cauchy formula evaluates interior values.

A simple-pole residue is $\lim_{z \rightarrow a} (z-a)f(z)$, and a positive contour integral is $2\pi i$ times enclosed residues.

Hinglish Explanation

Pehle singularities aur inside/outside check karo. Enclosed poles hi residue contribution dete hain.

4. Mathematical Formula Section

Formula Sheet

1. $z = x + iy$
2. $\bar{z} = x - iy$
3. $|z| = \sqrt{x^2 + y^2}$
4. $z = re^{i\theta}$
5. $\text{roots} = r^{1/n} e^{i(\theta + 2k\pi)/n}$
6. $u_x = v_y$
7. $u_y = -v_x$
8. $f' = u_x + i v_x$
9. $u_{xx} + u_{yy} = 0$
10. $\int_C f(z)/(z-a) dz = 2\pi i f(a)$
11. $\text{Res}(f, a) = \lim_{z \rightarrow a} (z-a)f(z)$

5. Formula Derivations and Solved Examples

De Moivre follows from $(re^{i\theta})^n = r^n e^{in\theta}$. Cube roots of unity have arguments $0, 2\pi/3, 4\pi/3$. For $u = x^2 - y^2$, CR gives $v = 2xy + C$ and $f = z^2 + iC$. Residue of $1/(z-a)$ is 1, so an enclosing positive contour integral is $2\pi i$.

6. Practical Applications

- AC phasors.
- Potential flow.
- Conformal mapping.
- Transform inversion.

7. Short Tricks

- Use polar for powers and roots.
- Plot roots equally spaced.
- Integrate CR systematically.
- For simple rational poles use numerator/denominator derivative.

8. Concept Diagram

$$\text{Im } z \quad z = (x, y)$$

$$|z| = r$$

$$\text{-----} + \text{-----} \rightarrow \text{Re } z$$

Conjugate reflects across real axis.

9. Comparison Table

Property	Real calculus	Complex analysis
Directions	line	all plane directions
Condition	real derivative	CR plus regularity
Components	one	harmonic pair

10. Common Mistakes

- Wrong argument quadrant.
- Listing one root only.
- CR at one point claimed analytic.
- Ignoring contour orientation.

11. Important DSSSB Exam Points

- Modulus-argument is foundational.
- CR and harmonicity are central.
- Mappings and simple residues are expected fundamentals.

12. Chapter Revision Sheet

Use Cartesian form for addition and polar form for products, powers and roots. Verify CR regularity. Identify poles and contour orientation before residues.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. $|3+4i|$

- (A) 3
- (B) 4
- (C) 5
- (D) 7

Correct Answer: (C)

Pythagoras.

Q2. Conjugate $2-3i$

- (A) $-2+3i$
- (B) $2+3i$
- (C) $-2-3i$
- (D) $3+2i$

Correct Answer: (B)

Flip imaginary sign.

Q3. i^2

- (A) 1
- (B) -1
- (C) i
- (D) $-i$

Correct Answer: (B)

Definition.

Q4. $\text{Arg}(-1)$

- (A) 0
- (B) $\pi/2$
- (C) π
- (D) 2π

Correct Answer: (C)

Principal argument.

Q5. $|zw|$

- (A) sum
- (B) product
- (C) difference
- (D) 1

Correct Answer: (B)

Multiplicative.

Q6. Arguments multiply by product?

- (A) add
- (B) multiply
- (C) vanish
- (D) divide

Correct Answer: (A)

Angles add.

Q7. Cube roots count

- (A) 1
- (B) 2
- (C) 3
- (D) infinite

Correct Answer: (C)

Degree three.

Q8. n th roots lie

- (A) line
- (B) equally spaced circle
- (C) parabola
- (D) random

Correct Answer: (B)

Equal modulus.

Q9. CR first

- (A) $u_x = v_y$
- (B) $u_x = -v_y$
- (C) $u_x = v_x$
- (D) $u_y = v_y$

Correct Answer: (A)

CR equation.

Q10. CR second

- (A) $u_y = v_x$
- (B) $u_y = -v_x$
- (C) $u_x = -v_x$
- (D) $u_y = v_y$

Correct Answer: (B)

Negative sign.

Q11. Analytic real part

- (A) harmonic
- (B) constant
- (C) discontinuous
- (D) imaginary

Correct Answer: (A)

Laplacian zero.

Q12. Derivative z^2

- (A) z
- (B) $2z$
- (C) z^2
- (D) 2

Correct Answer: (B)

Power rule.

Q13. conjugate z analytic

- (A) everywhere
- (B) no open set
- (C) outside origin
- (D) unit disk

Correct Answer: (B)

Direction dependent.

Q14. $w=z+a$

- (A) translation
- (B) inversion
- (C) square
- (D) reflection

Correct Answer: (A)

Shift.

Q15. $w=1/z$ modulus

- (A) r
- (B) r^2
- (C) $1/r$
- (D) $-r$

Correct Answer: (C)

Inversion.

Q16. Mobius maps circles/lines to

- (A) generalized circles
- (B) ellipses
- (C) points
- (D) lines only

Correct Answer: (A)

Standard theorem.

Q17. Closed analytic contour integral

- (A) 1
- (B) 0
- (C) $2\pi i$
- (D) infinity

Correct Answer: (B)

Cauchy theorem.

Q18. Residue $1/(z-a)$

- (A) 0
- (B) 1
- (C) a
- (D) infinity

Correct Answer: (B)

Limit.

Q19. Enclosing integral $1/(z-a)$

- (A) 0
- (B) πi
- (C) $2\pi i$
- (D) 1

Correct Answer: (C)

Residue theorem.

Q20. Conjugate of x^2-y^2

- (A) x^2+y^2
- (B) $2xy$
- (C) $x+y$
- (D) $-2xy$

Correct Answer: (B)

CR equations.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready for the exam only when its symbols, SI units, assumptions and sign convention are clear.