DSSSB AE/JE 2026

COMPLETE ENGINEERING MATHEMATICS

Complete Theory • Detailed Hinglish Explanation • Worked Examples & Shortcuts
Solved Examples • Practical Applications • Practice Questions

Prepared By

CIVILGYAN WITH SATISH

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Accuracy note: Mathematical results are stated with their conditions. Worked examples use exam-level notation and every numerical or optimization result should be verified in the original problem.

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Preface

This book develops engineering mathematics from vectors and calculus through probability, statistics, difference equations and linear programming. The sequence connects intuition, formulas, derivations, graphs, applications and exam decisions.

The scope follows the five prescribed DSSSB AE/JE chapters. Advanced detail is included only where it clarifies a listed theorem, distribution, recurrence or optimization method.

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How to Use This Book

Read the technical explanation first, then the Hinglish Explanation. Reproduce each derivation and graph, solve examples without viewing the result, and attempt all twenty questions at the end of every chapter.

For every answer, verify domain, sign, theorem conditions, probability range, statistical denominator, recurrence substitution and feasibility. Maintain a short error log and revise the chapter formula sheet after one day, one week and before the examination.

Practice rule: Questions are original DSSSB-style practice. Each solution explains only the correct answer, as requested.

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Table of Contents

How to use this book: Read Complete Theory, reinforce it with the Hinglish Explanation, review every formula and derivation row, then attempt all twenty questions without viewing the solutions. Questions are original DSSSB-style practice, not claimed verbatim previous-year questions.

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1. Vector Algebra and Vector Differential Calculus	Introduction • Complete Theory • Hinglish • Formulae and Derivations • Numericals • Applications • Diagram • Comparison • Exam Points • 20 Solved MCQs
2. Higher Differential and Integral Calculus	Introduction • Complete Theory • Hinglish • Formulae and Derivations • Numericals • Applications • Diagram • Comparison • Exam Points • 20 Solved MCQs
3. Probability Theory and Random Variables	Introduction • Complete Theory • Hinglish • Formulae and Derivations • Numericals • Applications • Diagram • Comparison • Exam Points • 20 Solved MCQs
4. Descriptive Statistics, Probability Distributions and Statistical Analysis	Introduction • Complete Theory • Hinglish • Formulae and Derivations • Numericals • Applications • Diagram • Comparison • Exam Points • 20 Solved MCQs
5. Difference Equations and Linear Programming	Introduction • Complete Theory • Hinglish • Formulae and Derivations • Numericals • Applications • Diagram • Comparison • Exam Points • 20 Solved MCQs

Mathematical Notation and Core Shortcuts

1. Use radians in calculus unless stated
2. distinguish vectors from scalars
3. write theorem conditions before applying results
4. probabilities lie from 0 to 1
5. variance is nonnegative
6. verify recurrence and LPP answers in the original conditions.

CHAPTER 1 — VECTOR ALGEBRA AND VECTOR DIFFERENTIAL CALCULUS

1. Introduction

Vectors describe magnitude and direction, while scalar and vector fields describe quantities distributed through space. Vector algebra combines forces and displacements; gradient, divergence and curl convert local field variation into geometric and physical information used in fluid flow, heat transfer and electromagnetism.

2. Complete Theory

Scalars, vectors and Cartesian components

A scalar is specified by magnitude alone, whereas a vector requires magnitude and direction. In Cartesian coordinates $a=a_x\, i+a_y\, j+a_z\, k$ and $|a|=\sqrt{(a_x^2+a_y^2+a_z^2)}$.

A unit vector along a is a/|a| when a is nonzero. The vector from point P to Q is r_Q-r_P.

Direction cosines l,m,n are cosines of angles with positive axes and satisfy $l^2+m^2+n^2=1$. Vectors add componentwise; the triangle and parallelogram laws give the same resultant geometrically.

Multiplication by a negative scalar reverses direction. Linear dependence means one vector combination can equal zero with coefficients not all zero.

Hinglish Explanation

Vector ko arrow samjho: length magnitude, arrowhead direction. Components x,y,z directions mein contributions hain.

P se Q vector nikalne ke liye final minus initial. Unit vector banane ke liye magnitude se divide karo; zero vector ko normalize nahi kar sakte.

Dot, cross and triple products

The dot product $a\cdot b=|a||b|\cos\theta$ is scalar and measures alignment. It gives work, angle and scalar projection.

Perpendicular nonzero vectors have zero dot product. The cross product $a\times b$ is a vector perpendicular to their plane, with magnitude $|a||b|\sin\theta$ and right-hand-rule direction; it gives moment and parallelogram area.

Order matters because $b\times a=-(a\times b)$. The scalar triple product $a\cdot(b\times c)$ is the signed parallelepiped volume and vanishes for coplanar vectors.

The vector triple product obeys $a\times(b\times c)=b(a\cdot c)-c(a\cdot b)$, not an associative rule.

Hinglish Explanation

Dot ka answer scalar aur alignment; cross ka answer perpendicular vector aur area/moment. Cross order reverse karoge to sign reverse.

Scalar triple product ka absolute value volume; zero aaye to vectors coplanar.

Scalar fields, vector fields and gradient

A scalar field $\varphi(x,y,z)$ assigns one number to every point, such as temperature. A vector field F assigns a vector, such as velocity.

Partial derivatives measure change with one coordinate while others remain fixed. The gradient $\nabla\varphi=(\partial\varphi/\partial x)i+(\partial\varphi/\partial y)j+(\partial\varphi/\partial z)k$ points in the direction of maximum increase; its magnitude is the maximum directional derivative.

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The directional derivative along unit u is $\nabla\varphi\cdot u$. Gradient is normal to a regular level surface φ=constant because motion tangent to that surface produces zero first-order change.

Hinglish Explanation

Temperature map scalar field, fluid velocity vector field. Gradient woh arrow hai jahan temperature fastest increase hota hai.

Level curve par move karne se value same, isliye tangent direction ka gradient ke saath dot product zero.

Divergence and curl

For F=P i+Q j+R k, divergence is $\nabla\cdot F=P_x+Q_y+R_z$. It is scalar and represents local net outward flux density: positive suggests a source and negative a sink.

Curl is $\nabla\times F$ and represents local rotational tendency; its direction follows the right-hand rule. A solenoidal field has zero divergence.

An irrotational field has zero curl. Identities include $\nabla\times(\nabla\varphi)=0$ and $\nabla\cdot(\nabla\times F)=0$ when derivatives are sufficiently smooth.

Zero curl on a suitable simply connected domain supports a scalar potential $F=\nabla\varphi$.

Hinglish Explanation

Divergence tiny box se net outward flow check karta hai. Curl tiny paddle wheel ki rotation.

'Zero curl means potential' bolne se pehle domain condition yaad rakho; hole wale domain mein complication ho sakti hai.

Line integrals and conservative fields

A scalar line integral $\int_C f\,ds$ accumulates density along length. Work along a curve is $\int_C F\cdot dr$.

Parameterize r(t), so dr=r'(t)dt and preserve orientation. If $F=\nabla\varphi$, the fundamental theorem gives $\int_C F\cdot dr=\varphi(B)-\varphi(A)$, independent of path.

Therefore a closed integral is zero in a conservative region. To find a potential, integrate one component and determine missing functions from the other components.

Path independence, zero closed circulation and potential existence are equivalent only under suitable domain and regularity assumptions.

Hinglish Explanation

Curve ko parameter t mein likho. Conservative field mein poora path integrate karne ki zarurat nahi: endpoint potential difference enough.

Closed loop ka answer zero, provided field/domain conditions valid hain.

Surface, volume integrals and integral theorems

Flux through an oriented surface is $\iint_S F\cdot n\,dS$. Reversing orientation changes sign.

Gauss divergence theorem converts outward flux over a closed surface into $\iiint_V \nabla\cdot F\,dV$. Stokes theorem converts circulation around a positively oriented boundary into surface integral of curl.

Green theorem is the plane version relating a closed line integral to a double integral over its region. These theorems require smooth fields, appropriate regions and consistent orientation; an open surface cannot be used directly as the closed surface in Gauss theorem.

Hinglish Explanation

Theorems boundary aur interior ko connect karte hain. Gauss: closed surface flux ↔ volume divergence.

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Stokes: boundary circulation ↔ surface curl. Arrow/orientation wrong hui to sign wrong.

4. Formula Section

Formula Sheet
1. $\lvert a\rvert=\sqrt{(a_x^2+a_y^2+a_z^2)}$
2. unit vector=$a/\lvert a\rvert$
3. $a\cdot b=\lvert a\rvert\lvert b\rvert\cos\theta$
4. $\lvert a\times b\rvert=\lvert a\rvert\lvert b\rvert\sin\theta$
5. volume=$\lvert a\cdot(b\times c)\rvert$
6. $\nabla\varphi=\varphi_x\, i+\varphi_y\, j+\varphi_z\, k$
7. directional derivative=$\nabla\varphi\cdot u$
8. $\nabla\cdot F=P_x+Q_y+R_z$
9. $\nabla\times F=\det[(i,j,k),(\partial_x,\partial_y,\partial_z),(P,Q,R)]$
10. $\int_C\nabla\varphi\cdot dr=\varphi(B)-\varphi(A)$
11. $\iint_S F\cdot n\, dS=\iiint_V\nabla\cdot F\, dV$
12. $\oint_C F\cdot dr=\iint_S(\nabla\times F)\cdot n\, dS$

5. Derivations and Solved Examples

For $\varphi=x^2y+z^2$, $\nabla\varphi=(2xy)i+x^2j+2zk$. At (1,2,1), gradient=(4,1,2) and the maximum directional derivative is $\sqrt{21}$. For F=(x,y,z), divergence is 3. On the sphere of radius a, Gauss gives outward flux $\iiint_{V3dV}=3(4\pi a^3/3)=4\pi a^3$. For a=(1,0,0), b=(0,2,0), $\lvert a\times b\rvert=2$, the parallelogram area.

6. Practical Applications

- Force resolution and moments.
- Heat-flow normals and temperature gradients.
- Fluid source, circulation and flux calculations.
- Electromagnetic field laws.
- Surveying and three-dimensional geometry.

7. Short Tricks

- Final minus initial for displacement.
- Dot zero tests perpendicularity.
- Cross magnitude gives parallelogram area.

• Gradient is normal to level surface.

8. Concept Diagram

FIELD MAP
level curves φ=c1,c2,c3
↑ ∇φ (normal, fastest increase)
source field: arrows outward => div F>0
rotation field: circulating arrows => curl F≠0
Gauss links closed flux to volume divergence; Stokes links boundary circulation to surface curl.

9. Comparison Table

Concept	Result type	Meaning
Dot product	scalar	alignment/work
Cross product	vector	normal/area/moment
Gradient	vector	scalar-field steepest rise
Divergence	scalar	net outward density
Curl	vector	local rotation

10. Common Mistakes

• Using a non-unit direction in directional derivative.

• Reversing cross-product order without sign change.

• Applying Gauss to an open surface.

• Ignoring curve or surface orientation.

• Claiming zero curl always implies global potential.

11. Important DSSSB Exam Points

• Dot/cross products, gradient direction, divergence/curl computations and Gauss/Stokes identification are high-yield.

12. Chapter Revision Sheet

Identify whether the answer must be scalar or vector. Normalize directions, state orientation, compute derivatives componentwise and check theorem conditions before replacing an integral.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. A scalar quantity has

(A) magnitude only
(B) r_Q-r_P
(C) coplanar vectors
(D) zero divergence

Correct Answer: (A)
A scalar has no required direction.

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Q2. The vector from P to Q is

(A) $l^2+m^2+n^2=1$
(B) r_Q-r_P
(C) vector
(D) zero curl

Correct Answer: (B)
Displacement is final position minus initial.

Q3. Direction cosines satisfy

(A) zero
(B) level surface
(C) $l^2+m^2+n^2=1$
(D) $\int_C F\cdot dr$

Correct Answer: (C)
They are components of a unit direction.

Q4. For perpendicular nonzero vectors, the dot product is

(A) parallelogram area
(B) $|\nabla\varphi|$
(C) endpoints only
(D) zero

Correct Answer: (D)
$\cos 90° = 0$.

Q5. The magnitude of a×b gives

(A) parallelogram area
(B) reverses the sign
(C) scalar
(D) closed surface

Correct Answer: (A)
It equals $|a||b|\sin\theta$.

Q6. Interchanging a and b in a cross product

(A) volume
(B) reverses the sign
(C) vector
(D) surface integral of curl

Correct Answer: (B)
$b\times a=-(a\times b)$.

Q7. The scalar triple product measures signed

(A) coplanar vectors
(B) zero divergence
(C) volume
(D) changes flux sign

Correct Answer: (C)
Its absolute value is parallelepiped volume.

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Q8. A zero scalar triple product indicates

(A) vector
(B) zero curl
(C) magnitude only
(D) coplanar vectors

Correct Answer: (D)
The spanned volume vanishes.

Q9. Gradient of a scalar field is a

(A) vector
(B) level surface
(C) $\int_C F \cdot dr$
(D) $r_Q - r_P$

Correct Answer: (A)
It contains the coordinate partial derivatives.

Q10. Gradient is normal to a

(A) $|\nabla\varphi|$
(B) level surface
(C) endpoints only
(D) $l^2+m^2+n^2=1$

Correct Answer: (B)
Tangential motion has zero directional derivative.

Q11. Maximum directional derivative equals

(A) scalar
(B) closed surface
(C) $|\nabla\varphi|$
(D) zero

Correct Answer: (C)
It occurs along the gradient unit direction.

Q12. Divergence of a vector field is a

(A) vector
(B) surface integral of curl
(C) parallelogram area
(D) scalar

Correct Answer: (D)
It measures net outward flux density.

Q13. Curl of a vector field is a

(A) vector
(B) zero divergence
(C) changes flux sign
(D) reverses the sign

Correct Answer: (A)
It measures rotational tendency and axis.

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Q14. A solenoidal field has

(A) zero curl
(B) zero divergence
(C) magnitude only
(D) volume

Correct Answer: (B)
That is the defining condition.

Q15. An irrotational field has

(A) $\int_C$ F·dr
(B) r_Q-r_P
(C) zero curl
(D) coplanar vectors

Correct Answer: (C)
That is the defining condition.

Q16. Work along C is

(A) endpoints only
(B) $l^2+m^2+n^2=1$
(C) vector
(D) $\int_C$ F·dr

Correct Answer: (D)
The field is dotted with displacement.

Q17. A conservative line integral depends on

(A) endpoints only
(B) closed surface
(C) zero
(D) level surface

Correct Answer: (A)
It equals the potential difference.

Q18. Gauss theorem requires a

(A) surface integral of curl
(B) closed surface
(C) parallelogram area
(D) $|\nabla\varphi|$

Correct Answer: (B)
It relates its outward flux to volume divergence.

Q19. Stokes theorem relates circulation to

(A) changes flux sign
(B) reverses the sign
(C) surface integral of curl
(D) scalar

Correct Answer: (C)
Boundary and surface orientations must agree.

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Q20. Reversing surface orientation

(A) magnitude only
(B) volume
(C) vector
(D) changes flux sign

Correct Answer: (D)

The unit normal reverses.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready only when its symbols, domain, assumptions and validity conditions are clear.

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CHAPTER 2 — HIGHER DIFFERENTIAL AND INTEGRAL CALCULUS

1. Introduction

Higher calculus formalizes limits, smooth change, approximation, multivariable sensitivity and accumulation over regions. The central habit is to state domain and theorem conditions, then use geometry to choose derivatives, coordinates and integration limits.

2. Complete Theory

Functions, limits, continuity and differentiability

A function maps each allowed input to one output. The domain controls valid algebra and graphs.

A limit describes approached behaviour; continuity at a requires left limit, right limit and f(a) to agree. Differentiability means a finite common tangent slope and implies continuity, but a continuous function such as |x| need not be differentiable at a corner.

Indeterminate forms such as 0/0 request simplification, not the answer zero. L'Hospital's rule applies only to eligible indeterminate forms under its hypotheses.

For multivariable functions, path dependence can destroy a limit.

Hinglish Explanation

Limit point ke paas behaviour, continuity mein actual value bhi same. Smooth tangent differentiability hai.

0/0 ko answer mat likho; factor, rationalize ya standard limit use karo. Two-variable limit mein different paths check karna pad sakta hai.

Mean-value theorems and Taylor expansion

Rolle theorem requires continuity on [a,b], differentiability on (a,b) and f(a)=f(b), then some c has f'(c)=0. Lagrange MVT removes equal endpoints and gives f'(c)=[f(b)-f(a)]/(b-a).

Cauchy MVT compares two functions with additional denominator conditions. Taylor's theorem represents a smooth function near a by derivatives at a plus a remainder; Maclaurin uses a=0.

The remainder determines approximation error, and a formal series is not automatically valid outside its convergence interval.

Hinglish Explanation

Theorem formula se pehle conditions tick karo. MVT graphically endpoint secant ke parallel interior tangent guarantee karta hai.

Taylor function ka local polynomial model hai; remainder accuracy batata hai.

Partial derivatives, total differential and chain rule

For z=f(x,y), f_x varies x while holding y fixed. Mixed partials agree under suitable continuity.

The total differential $dz=f_{xdx}+f_{ydy}$ is the first-order combined change and estimates propagated error. If x and y depend on t, $dz/dt=f_x\ dx/dt+f_y\ dy/dt$.

For composite variables u,v, the multivariable chain rule sums every dependency path. A function homogeneous of degree n satisfies Euler's relation $xf_x+yf_y=nf$.

Dimensional scaling often reveals homogeneity.

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Hinglish Explanation

Partial derivative mein ek knob move, doosra freeze. Total differential sab small changes add karta hai.

Chain rule ko dependency tree ki har path se contribution samjho. Euler theorem scaling degree ko derivatives se connect karta hai.

Maxima, minima and constrained optimization

For an interior extremum of f(x,y), stationary candidates satisfy $f_x=f_y=0$. The Hessian determinant $D=f_{xx} f_{yy}-f_{xy}^2$ classifies: D>0 with $f_{xx}>0$ gives local minimum; D>0 with $f_{xx}<0$ maximum; D<0 saddle; D=0 is inconclusive.

Boundaries and nondifferentiable points must also be tested for global extrema. Under a smooth constraint g=0, Lagrange multipliers solve $\nabla f=\lambda\nabla g$ together with g=0, provided constraint regularity holds.

The multiplier measures marginal objective change with the constraint level.

Hinglish Explanation

Stationary point hill, bowl ya saddle ho sakta hai; Hessian classify karta hai. D=0 par forced conclusion nahi.

Constraint ek fence hai, optimum par objective contour aur constraint tangent hote hain.

Multiple integrals and coordinate transformations

A double integral sums over area and a triple integral over volume. Iterated limits must describe the region exactly; changing order requires redrawing it.

Polar coordinates $x=r\cos\theta, y=r\sin\theta$ carry Jacobian r, so $dA=r\,dr d\theta$. Cylindrical volume also carries r; spherical volume carries $\rho^2\sin\varphi$.

A general substitution uses the absolute Jacobian determinant. Symmetry can eliminate odd contributions or reduce the region.

Mass integrates density, and centroid equals first moment divided by total mass.

Hinglish Explanation

Limits ratne se pehle region draw karo. Coordinate change mein area tile stretch hoti hai; Jacobian compulsory correction factor.

Circle ke liye polar natural, but r factor bhoolna common error.

Improper integrals and engineering applications

An improper integral has an infinite interval or unbounded integrand and is defined by a limit. Convergence must be established before algebraic manipulation.

Comparison tests relate a nonnegative integrand to a known convergent or divergent one. The p-integral $\int_1^\infty x^{-p}dx$ converges only for p>1.

Definite integrals compute area, volume, work and accumulated flow. Differentiation under an integral sign or exchanging integrals requires relevant regularity and convergence; exam problems usually state or imply manageable conditions.

Hinglish Explanation

Infinity ko upper limit number jaise substitute mat karo; limit define karo. p-test mein p>1 convergence.

Engineering accumulation ka sign aur units final sanity check dete hain.

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4. Formula Section

Formula Sheet
1. continuity at a: $\lim_{x\to a} f(x)=f(a)$
2. MVT $f'(c)=[f(b)-f(a)]/(b-a)$
3. Taylor $f(x)=\Sigma\, f^{(n)}(a)(x-a)^n/n!$
4. $e^x=1+x+x^2/2!+\ldots$
5. $\sin x=x-x^3/3!+\ldots$
6. $dz=f_{xdx}+f_{ydy}$
7. $xf_x+yf_y=nf$
8. $D=f_{xx}\, f_{yy}-f_{xy}{}^2$
9. $\nabla f=\lambda\nabla g$
10. $dA_{polar}=r\, dr d\theta$
11. $dV_{spherical}=\rho^2\sin\varphi\, d\rho d\varphi d\theta$
12. centroid $\bar{x}=(1/M)\int x\, dm$
13. $\int_1$^$\infty x^{-p}dx$ converges iff $p>1$

5. Derivations and Solved Examples

For $f=x^2+y^2-2x-4y$, $f_x=2x-2$ and $f_y=2y-4$ give (1,2). Here $D=4>0$ and $f_{xx}=2>0$, so it is a minimum with value -5. For the unit disk, area$=\int_0^{2\pi}\int_0^1 r\, dr d\theta=\pi$. Taylor gives $e^{0.1}\approx1+0.1+0.1^2/2=1.105$, close to the exact value.

6. Practical Applications

• Structural and design optimization.

• Error propagation in measurements.

• Area, volume and centroid calculation.

• Heat and flow accumulation.

• Series linearization of nonlinear models.

7. Short Tricks

• Differentiability implies continuity only one way.

• Draw the region before limits.

• Use symmetry before integration.

• $D<0$ means saddle.

8. Concept Diagram

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CALCULUS MAP
limit -> continuity -> derivative -> local linear model
                        -> Taylor higher-order model
f(x,y): partial slopes -> gradient/Hessian -> constrained optimum
region sketch -> coordinate choice -> Jacobian -> multiple integral
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9. Comparison Table

Concept	Condition/result	Trap
Continuity	limit=value	does not imply differentiability
Differentiability	finite common slope	implies continuity
Local extremum	stationary candidate	boundary still matters
Polar integral	$dA=r\,dr d\theta$	never omit r

10. Common Mistakes

• Applying a theorem without endpoint conditions.

• Calling $D=0$ a maximum or minimum.

• Dropping chain-rule paths.

• Omitting a Jacobian.

• Treating ∞ as a number.

11. Important DSSSB Exam Points

• Standard limits, MVT conditions, Taylor coefficients, Euler theorem, Hessian and polar integrals are frequent.

12. Chapter Revision Sheet

Start with domain and smoothness. Check theorem conditions, derive candidates, inspect boundaries, draw integration regions and verify signs, units and limiting behaviour.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. Differentiability implies

(A) continuity
(B) differentiability
(C) $f_{xdx}+f_{ydy}$
(D) area

Correct Answer: (A)
A differentiable function is continuous.

Q2. Continuity alone does not guarantee

(A) $f(a)=f(b)$
(B) differentiability
(C) $xf_x+yf_y=nf$
(D) volume

Correct Answer: (B)
A corner such as $|x|$ is continuous but not differentiable.

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Q3. Rolle theorem additionally requires

(A) the endpoint secant slope
(B) $f_x=f_y=0$
(C) f(a)=f(b)
(D) r drdθ

Correct Answer: (C)
Equal endpoint values yield a horizontal interior tangent.

Q4. Lagrange MVT gives an interior derivative equal to

(A) 0
(B) a saddle point
(C) $\rho^2\sin\varphi$
(D) the endpoint secant slope

Correct Answer: (D)
That slope is [f(b)-f(a)]/(b-a).

Q5. Maclaurin series is Taylor series about

(A) 0
(B) 1/2!
(C) inconclusive
(D) first moment divided by mass

Correct Answer: (A)
Maclaurin uses a=0.

Q6. The x^2 coefficient in e^x is

(A) other independent variables fixed
(B) 1/2!
(C) Lagrange multipliers
(D) p>1

Correct Answer: (B)
The nth coefficient is 1/n!.

Q7. A partial derivative f_x holds

(A) $f_{xdx}+f_{ydy}$
(B) area
(C) other independent variables fixed
(D) a limit

Correct Answer: (C)
It measures one-coordinate sensitivity.

Q8. The first-order total differential is

(A) $xf_x+yf_y=nf$
(B) volume
(C) continuity
(D) $f_{xdx}+f_{ydy}$

Correct Answer: (D)
It combines coordinate changes.

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Q9. Euler theorem for degree n is

(A) $xf_x+yf_y=nf$
(B) $f_x=f_y=0$
(C) $r\,drd\theta$
(D) differentiability

Correct Answer: (A)
It characterizes differentiable homogeneous functions.

Q10. An interior stationary point satisfies

(A) a saddle point
(B) $f_x=f_y=0$
(C) $\rho^2\sin\varphi$
(D) f(a)=f(b)

Correct Answer: (B)
These are necessary first-order conditions.

Q11. D<0 in the two-variable Hessian test indicates

(A) inconclusive
(B) first moment divided by mass
(C) a saddle point
(D) the endpoint secant slope

Correct Answer: (C)
Curvature has opposite signs.

Q12. D=0 in the Hessian test is

(A) Lagrange multipliers
(B) p>1
(C) 0
(D) inconclusive

Correct Answer: (D)
Higher-order analysis may be needed.

Q13. A constrained optimum commonly uses

(A) Lagrange multipliers
(B) area
(C) a limit
(D) 1/2!

Correct Answer: (A)
It enforces gradient parallelism.

Q14. A double integral of 1 over a region gives

(A) volume
(B) area
(C) continuity
(D) other independent variables fixed

Correct Answer: (B)
It sums differential area.

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Q15. A triple integral of 1 over a solid gives

(A) r drdθ
(B) differentiability
(C) volume
(D) $f_{xdx}+f_{ydy}$

Correct Answer: (C)
It sums differential volume.

Q16. Polar area element is

(A) $\rho^2\sin\varphi$
(B) f(a)=f(b)
(C) $xf_x+yf_y=nf$
(D) r drdθ

Correct Answer: (D)
The Jacobian is r.

Q17. Spherical volume element contains

(A) $\rho^2\sin\varphi$
(B) first moment divided by mass
(C) the endpoint secant slope
(D) $f_x=f_y=0$

Correct Answer: (A)
This is the spherical Jacobian.

Q18. Centroid is calculated as

(A) p>1
(B) first moment divided by mass
(C) 0
(D) a saddle point

Correct Answer: (B)
It is a weighted average position.

Q19. The integral $\int_1$^∞x^{-p}dx converges for

(A) a limit
(B) 1/2!
(C) p>1
(D) inconclusive

Correct Answer: (C)
This is the p-integral criterion.

Q20. An improper integral is defined through

(A) continuity
(B) other independent variables fixed
(C) Lagrange multipliers
(D) a limit

Correct Answer: (D)
Infinite bounds or singularities require limiting definitions.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready only when its symbols, domain, assumptions and validity conditions are clear.

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CHAPTER 3 — PROBABILITY THEORY AND RANDOM VARIABLES

1. Introduction

Probability quantifies uncertainty using events in a sample space. Random variables turn outcomes into numbers, and distributions describe their probabilities. Careful distinction between mutually exclusive and independent events, and between PMF, PDF and CDF, prevents most objective-question errors.

2. Complete Theory

Sample spaces, events and counting

A sample space S contains all possible outcomes and an event is a subset. Probability axioms give P(S)=1, nonnegativity and additivity for disjoint events.

Complements satisfy $P(A^c)=1-P(A)$. Counting uses permutations when order matters and combinations when order does not.

The multiplication principle handles sequential choices. Equally likely probability is favourable outcomes divided by total outcomes only when the underlying outcomes truly have equal probability.

Hinglish Explanation

Pehle experiment ka sample space define karo. Arrangement mein order matter to permutation; selection mein order irrelevant to combination.

Favourable/total formula blindly use mat karo unless outcomes equally likely.

Addition, conditional probability and Bayes theorem

The addition law is $P(A∪B)=P(A)+P(B)-P(A∩B)$. Mutually exclusive events have empty intersection.

Conditional probability $P(A|B)=P(A∩B)/P(B)$ updates probability after B occurs. Independence means $P(A∩B)=P(A)P(B)$, equivalently $P(A|B)=P(A)$ when denominators are valid.

Mutually exclusive nonzero events are not independent. Total probability partitions causes, and Bayes theorem reverses conditioning: posterior is likelihood times prior divided by evidence.

Hinglish Explanation

Exclusive ka matlab saath nahi ho sakte; independent ka matlab ek event doosre ki probability change nahi karta. Bayes mein direction reverse hoti hai, P(A|B) ko P(B|A) mat samjho.

Discrete and continuous random variables

A random variable X assigns a real number to each outcome. A discrete PMF $p(x)=P(X=x)$ is nonnegative and sums to one.

A continuous PDF f(x) is nonnegative and integrates to one, but f(x) itself may exceed one because probability is area; $P(X=x)=0$ at a single point. The CDF $F(x)=P(X≤x)$ is nondecreasing, right-continuous and approaches 0 and 1 at the extremes.

For continuous X, probabilities are integrals and endpoint inclusion does not matter.

Hinglish Explanation

PMF point probability deta hai; PDF height probability nahi, area probability hai. Continuous variable ke exact point ki probability zero ho sakti hai even though PDF positive.

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CDF hamesha cumulative aur nondecreasing.

Expectation, variance and transformations

Expectation is the probability-weighted average: $E[X]=\Sigma xp(x)$ or $\int xf(x)dx$. Linearity gives $E[aX+b]=aE[X]+b$ without independence.

Variance $Var(X)=E[(X-\mu)^2]=E[X^2]-\mu^2$ and is nonnegative. $Var(aX+b)=a^2Var(X)$.

For independent X,Y, variances add in a sum; covariance is zero, although zero covariance alone does not generally imply independence. A transformation $Y=g(X)$ can be handled through its CDF or change-of-variable density with all inverse branches.

Hinglish Explanation

Mean long-run centre, variance squared spread. Constant add karne se variance change nahi; scaling a se variance a^2 times.

Independent sums mein variance add hoti hai.

Joint, marginal and conditional distributions

A joint distribution describes two variables simultaneously. Marginals are obtained by summing or integrating over the other variable.

Conditional distributions divide joint probability or density by the appropriate marginal. Independence requires joint factorization into marginals over the support.

Covariance $E[(X-\mu_X)(Y-\mu_Y)]$ measures linear co-variation; correlation normalizes it to [-1,1] when standard deviations are nonzero. Correlation near zero does not rule out nonlinear dependence.

Hinglish Explanation

Joint table se row/column totals marginals. Conditional ke liye relevant marginal se divide.

Independent hone par joint=marginal product. Zero correlation ko automatic independence mat bolo.

Reliability interpretation and common models

For independent components in series, the system works only if all work, so reliability is the product. A parallel system works if at least one works, so calculate one minus the probability all fail.

Independence must be justified; common environments can correlate failures. Bernoulli trials, waiting times and lifetime distributions connect probability to quality control, maintenance and project risk.

Models are approximations whose assumptions and support must match the physical quantity.

Hinglish Explanation

Series system chain jaisa: one failure stops system. Parallel redundancy mein all fail hona required, so 1-product of failure probabilities.

Common power supply failures independence assumption tod sakte hain.

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4. Formula Section

Formula Sheet
1. $P(A^c)=1-P(A)$
2. $nP_r=n!/(n-r)!$
3. $nC_r=n!/[r!(n-r)!]$
4. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
5. $P(A\|B)=P(A\cap B)/P(B)$
6. independence $P(A\cap B)=P(A)P(B)$
7. Bayes $P(A_i\|B)=P(B\|A_i)P(A_i)/\Sigma P(B\|A_j)P(A_j)$
8. $F(x)=P(X\le x)$
9. $E[X]=\Sigma xp(x)$ or $\int xf(x)dx$
10. $Var(X)=E[X^2]-[E(X)]^2$
11. $Var(aX+b)=a^2Var(X)$
12. $\rho=Cov(X,Y)/(\sigma_X\sigma_Y)$
13. $R_{series}=\prod R_i$
14. $R_{parallel}=1-\prod(1-R_i)$

5. Derivations and Solved Examples

If $P(A)=0.6$, $P(B)=0.5$ and $P(A\cap B)=0.3$, then A and B are independent because $0.6\times0.5=0.3$. For X with $P(0)=0.2,P(1)=0.5,P(2)=0.3$, $E[X]=1.1$ and $E[X^2]=1.7$, so $Var(X)=1.7-1.21=0.49$. Two independent parallel components of reliability 0.9 give $1-(0.1)^2=0.99$.

6. Practical Applications

- Reliability and redundancy.
- Quality and acceptance sampling.
- Hydrological risk.
- Project and cost uncertainty.
- Demand forecasting.

7. Short Tricks

- Use complements for 'at least one'.
- Bayes denominator is total evidence.

• CDF differences give interval probabilities.

• Variance cannot be negative.

8. Concept Diagram

EVENT RELATIONS
A∪B: at least one
A∩B: both
A^c: not A
conditioning B: restrict sample space to B
Bayes: prior -> likelihood/evidence -> posterior
CDF accumulates probability from -∞ to x.

9. Comparison Table

Pair	Meaning	Key test
Exclusive	cannot occur together	P(A∩B)=0
Independent	no probability influence	P(A∩B)=P(A)P(B)
PMF	discrete mass	sum=1
PDF	continuous density	integral=1
CDF	cumulative probability	nondecreasing

10. Common Mistakes

• Confusing P(A|B) with P(B|A).

• Calling exclusive events independent.

• Treating PDF height as point probability.

• Forgetting all inverse branches in transformations.

• Adding variances without independence/covariance analysis.

11. Important DSSSB Exam Points

• Addition law, Bayes tables, PMF/PDF/CDF, expectation-variance and reliability combinations are high-yield.

12. Chapter Revision Sheet

Define the sample space and event relation first. Normalize distributions, check probability bounds, compute expectation from support and verify reliability logic through complements.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. Probability of the sample space is

(A) 1
(B) 1-P(A)
(C) not independent
(D) nondecreasing

Correct Answer: (A)
This is an axiom.

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Q2. For any event, $P(A^C)$ equals

(A) permutations
(B) 1-P(A)
(C) P(B)
(D) a probability-weighted average

Correct Answer: (B)
Event and complement partition S.

Q3. When order matters, use

(A) combinations
(B) a posterior probability
(C) permutations
(D) $E[X^2]-[E(X)]^2$

Correct Answer: (C)
Arrangements distinguish order.

Q4. When order does not matter, use

(A) $P(A \cap B)$
(B) a discrete random variable
(C) does not change variance
(D) combinations

Correct Answer: (D)
Selections ignore order.

Q5. The addition law subtracts

(A) $P(A \cap B)$
(B) $P(A \cap B)=0$
(C) integration to one
(D) a^2

Correct Answer: (A)
The overlap was counted twice.

Q6. Mutually exclusive events have

(A) $P(A \cap B)=P(A)P(B)$
(B) $P(A \cap B)=0$
(C) 0
(D) -1 and 1

Correct Answer: (B)
They cannot occur simultaneously.

Q7. Independent events satisfy

(A) not independent
(B) nondecreasing
(C) $P(A \cap B)=P(A)P(B)$
(D) the product of component reliabilities

Correct Answer: (C)
Their joint probability factorizes.

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Q8. Nonzero mutually exclusive events are

(A) P(B)
(B) a probability-weighted average
(C) 1
(D) not independent

Correct Answer: (D)
Occurrence of one makes the other impossible.

Q9. Conditional probability P(A|B) has denominator

(A) P(B)
(B) a posterior probability
(C) $E[X^2]-[E(X)]^2$
(D) 1-P(A)

Correct Answer: (A)
The sample space is restricted to B.

Q10. Bayes theorem calculates

(A) a discrete random variable
(B) a posterior probability
(C) does not change variance
(D) permutations

Correct Answer: (B)
It reverses conditioning using priors and likelihoods.

Q11. A PMF is used for

(A) integration to one
(B) a^2
(C) a discrete random variable
(D) combinations

Correct Answer: (C)
It assigns point masses.

Q12. A PDF is normalized by

(A) 0
(B) -1 and 1
(C) $P(A\cap B)$
(D) integration to one

Correct Answer: (D)
Continuous probability is area.

Q13. For a continuous X, P(X=x) is

(A) 0
(B) nondecreasing
(C) the product of component reliabilities
(D) $P(A\cap B)=0$

Correct Answer: (A)
A single point has zero area.

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Q14. A CDF is

(A) a probability-weighted average
(B) nondecreasing
(C) 1
(D) $P(A \cap B)=P(A)P(B)$

Correct Answer: (B)
Accumulated probability cannot decrease.

Q15. Expectation is

(A) $E[X^2]-[E(X)]^2$
(B) 1-P(A)
(C) a probability-weighted average
(D) not independent

Correct Answer: (C)
It is the distribution centre.

Q16. Variance equals

(A) does not change variance
(B) permutations
(C) P(B)
(D) $E[X^2]-[E(X)]^2$

Correct Answer: (D)
This is the computational identity.

Q17. Adding a constant to X

(A) does not change variance
(B) a^2
(C) combinations
(D) a posterior probability

Correct Answer: (A)
Spread about the shifted mean is unchanged.

Q18. Multiplying X by a multiplies variance by

(A) -1 and 1
(B) a^2
(C) $P(A \cap B)$
(D) a discrete random variable

Correct Answer: (B)
Squared deviations scale by a^2.

Q19. Correlation always lies between

(A) the product of component reliabilities
(B) $P(A \cap B)=0$
(C) -1 and 1
(D) integration to one

Correct Answer: (C)
Cauchy-Schwarz gives the bound.

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Q20. Independent series reliability is

(A) 1
(B) $P(A \cap B)=P(A)P(B)$
(C) 0
(D) the product of component reliabilities

Correct Answer: (D)

Every series component must work.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready only when its symbols, domain, assumptions and validity conditions are clear.

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CHAPTER 4 — DESCRIPTIVE STATISTICS, PROBABILITY DISTRIBUTIONS AND STATISTICAL ANALYSIS

1. Introduction

Statistics summarizes data and supports inference about populations. Distribution models connect theory to observed counts or measurements. The exam-critical distinctions are parameter versus statistic, population versus sample denominators, correlation versus regression and Type I versus Type II errors.

2. Complete Theory

Data summaries and graphical displays

Population is the complete target set; a sample is observed subset. A parameter describes a population and a statistic is computed from a sample.

Data may be qualitative or quantitative, discrete or continuous. Frequency tables group observations.

Histograms use adjacent bars for continuous classes and area represents frequency when widths differ. A frequency polygon joins representative class points; an ogive plots cumulative frequency and supports median or percentile reading.

Misleading axes, unequal-width bars treated by height alone and selective ranges can distort interpretation.

Hinglish Explanation

Population poora target, sample uska observed part. Histogram continuous classes ke adjacent bars; unequal width mein frequency density important.

Ogive cumulative graph se median/percentile read kar sakte hain.

Mean, median, mode and dispersion

Arithmetic mean uses all values and is sensitive to extremes. Median is the middle ordered value and is robust to outliers.

Mode is most frequent and can be multiple or absent. Weighted mean uses specified weights.

Range is maximum minus minimum. Population variance divides squared deviations by N; sample variance commonly divides by n-1 for unbiased estimation under standard sampling.

Standard deviation is square root of variance and returns to original units. Coefficient of variation compares relative spread when means are positive and meaningful.

Hinglish Explanation

Mean extreme values se pull hota hai; median robust. Variance squared units, SD original units.

Population ke liye N, sample estimate ke liye n-1—question wording pehle read karo.

Binomial, Poisson and normal distributions

Binomial X counts successes in n independent Bernoulli trials with constant success probability p: $P(X=x)=nC_x\, p^x q^{n-x}$, mean np and variance npq. Poisson counts events with rate λ in a fixed exposure: $P(X=x)=e^{-\lambda}\lambda^x/x!$, mean and variance λ.

It approximates binomial for large n and small p with $np \approx \lambda$. Normal is continuous, symmetric and determined by μ, σ^2.

Standardization $z=(x-\mu)/\sigma$ converts to $N(0,1)$; normal probabilities are areas, and continuity correction improves discrete approximations.

Hinglish Explanation

Binomial fixed n trials, Poisson event count rate, normal continuous bell curve. Binomial mein trials independent and p constant.

Normal table use se pehle z-score sign aur tail area check karo.

Other distributions and moments

A uniform continuous distribution has constant density on $[a,b]$, mean $(a+b)/2$ and variance $(b-a)^2/12$. Exponential waiting time with rate λ has density $\lambda e^{-\lambda x}$ for $x \geq 0$, mean $1/\lambda$ and memoryless property.

Moments summarize shape: first central moment is zero, second is variance, standardized third relates skewness and fourth relates kurtosis. Positive skew has a longer right tail.

Distribution choice must match support: a normal model allows all real values, whereas lifetime cannot be negative.

Hinglish Explanation

Uniform mein interval ke equal-length parts equal probability. Exponential waiting/lifetime nonnegative aur memoryless.

Model support physical quantity se match hona chahiye.

Correlation and regression

Covariance sign indicates joint linear direction but depends on units. Pearson correlation r standardizes covariance, is unit-free and lies in $[-1,1]$.

It measures linear association, not causation. Regression predicts one variable from another.

The regression line of Y on X minimizes squared vertical residuals; the line of X on Y solves a different prediction problem. Both pass through $(\bar{x},\bar{y})$.

Their coefficients satisfy $b_{yx}\, b_{xy}=r^2$ and have the sign of r. Interchanging variables is not harmless.

Hinglish Explanation

Correlation association strength, causation proof nahi. Y on X line Y predict karti hai aur vertical errors minimize; X on Y different line.

Dono mean point se pass.

Sampling, confidence and hypothesis testing

A sampling distribution describes a statistic over repeated samples. Standard error of the sample mean is $\sigma/\sqrt{n}$ when population σ is known.

By the central limit theorem, suitably standardized sums or means approach normality under broad conditions as n grows. A confidence interval gives a procedure with stated long-run coverage, not a probability that a fixed computed parameter changes.

In testing, H0 is assessed against H1. Type I error rejects true H0 with probability α; Type II error fails to reject false H0 with probability β.

A p-value is the probability, assuming H0, of data at least as extreme as observed.

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Hinglish Explanation

Standard error sample size √n ke saath reduce. Type I false alarm, Type II missed detection.

p-value H0 true hone ki probability nahi; H0 assume karke observed-or-more-extreme data probability.

4. Formula Section

Formula Sheet
1. $\bar{x}=\Sigma x/n$
2. weighted mean$=\Sigma wx/\Sigma w$
3. population variance $\sigma^2=\Sigma(x-\mu)^2/N$
4. sample variance $s^2=\Sigma(x-\bar{x})^2/(n-1)$
5. SD=√variance
6. CV=SD/mean x100%
7. binomial $P(X=x)=nC_x\ p^x(1-p)^{n-x}$
8. E=np
9. Var=np(1-p)
10. Poisson $P(X=x)=e^{-\lambda}\lambda^x/x!$
11. $E=Var=\lambda$
12. $z=(x-\mu)/\sigma$
13. uniform mean=(a+b)/2
14. uniform variance$=(b-a)^2/12$
15. exponential mean$=1/\lambda$
16. $r=Cov(X,Y)/(\sigma_X\sigma_Y)$
17. $SE(\bar{x})=\sigma/\sqrt{n}$

5. Derivations and Solved Examples

For data 2,4,4,6, mean=4. Population variance=[4+0+0+4]/4=2 and population SD=√2. For X~Bin(10,0.2), E=2 and Var=1.6. For Poisson λ=3, P(X=0)=e^{-3}. A normal observation x=70 from μ=60,σ=5 has z=2. If r=-0.8, both regression coefficients are negative and their product is 0.64.

6. Practical Applications

- Quality control charts and tolerances.
- Traffic and demand analysis.
- Reliability and lifetime studies.
- Survey and hydrology data.
- Forecasting and calibration.

7. Short Tricks

- Mean=median=mode for ideal symmetric unimodal normal.
- Poisson mean equals variance.
- Regression lines meet at means.
- Type I is false rejection.

8. Concept Diagram

DISTRIBUTION SHAPES
Binomial: discrete finite count 0...n
Poisson: discrete count 0,1,2,...
Normal: symmetric continuous bell around μ
Exponential: right-skewed waiting time x≥0
Testing: truth x decision => Type I false rejection; Type II missed rejection.

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9. Comparison Table

Concept	Population	Sample
Mean	μ	x̄
Variance divisor	N	n-1 commonly
Descriptor	parameter	statistic
Spread units	σ original	s original

10. Common Mistakes

- Using n instead of n-1 without reading context.
- Using normal density height as probability.
- Confusing regression directions.
- Claiming correlation proves causation.
- Interpreting p-value as P(H0 true).

11. Important DSSSB Exam Points

- Central tendency, sample variance, binomial/Poisson parameters, z-scores, regression properties and testing errors are high-yield.

12. Chapter Revision Sheet

Identify population/sample and variable support. Choose the distribution by experiment structure, standardize carefully, and interpret inferential quantities in repeated-sampling terms.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. A population descriptor is a

(A) parameter
(B) statistic
(C) np
(D) (x-μ)/σ

Correct Answer: (A)
Parameters describe populations.

Q2. A sample descriptor is a

(A) extreme values
(B) statistic
(C) np(1-p)
(D) x≥0

Correct Answer: (B)
Statistics are computed from samples.

Q3. The mean is most sensitive to

(A) the middle ordered value
(B) λ
(C) extreme values
(D) 1/λ

Correct Answer: (C)
Every observation contributes numerically.

Q4. The median is

(A) the original data units
(B) n is large and p is small
(C) [-1,1]
(D) the middle ordered value

Correct Answer: (D)
It depends on order and is robust.

Q5. Standard deviation has

(A) the original data units
(B) n-1
(C) symmetric about μ
(D) association, not causation

Correct Answer: (A)
It is the square root of variance.

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Q6. The common unbiased sample-variance divisor is

(A) fixed independent trials with constant p
(B) n-1
(C) 0 and 1
(D) $(\bar{x},\bar{y})$

Correct Answer: (B)
One degree of freedom is used by $\bar{x}$.

Q7. A binomial model requires

(A) np
(B) $(x-\mu)/\sigma$
(C) fixed independent trials with constant p
(D) rejecting a true H0

Correct Answer: (C)
These are core Bernoulli-trial assumptions.

Q8. Binomial mean is

(A) np(1-p)
(B) $x \geq 0$
(C) parameter
(D) np

Correct Answer: (D)
It is the sum of n Bernoulli means.

Q9. Binomial variance is

(A) np(1-p)
(B) λ
(C) $1/\lambda$
(D) statistic

Correct Answer: (A)
Independent Bernoulli variances add.

Q10. Poisson mean and variance are both

(A) n is large and p is small
(B) λ
(C) [-1,1]
(D) extreme values

Correct Answer: (B)
This is a defining property.

Q11. Poisson can approximate binomial when

(A) symmetric about μ
(B) association, not causation
(C) n is large and p is small
(D) the middle ordered value

Correct Answer: (C)
Keep np approximately λ.

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Q12. A normal distribution is

(A) 0 and 1
(B) $(\bar{x},\bar{y})$
(C) the original data units
(D) symmetric about μ

Correct Answer: (D)
Its mean, median and mode coincide.

Q13. The standard normal has mean and variance

(A) 0 and 1
(B) (x-μ)/σ
(C) rejecting a true H0
(D) n-1

Correct Answer: (A)
Standardization creates N(0,1).

Q14. A z-score equals

(A) x≥0
(B) (x-μ)/σ
(C) parameter
(D) fixed independent trials with constant p

Correct Answer: (B)
It measures signed standard deviations.

Q15. Exponential support is

(A) 1/λ
(B) statistic
(C) x≥0
(D) np

Correct Answer: (C)
Waiting time cannot be negative.

Q16. Exponential mean is

(A) [-1,1]
(B) extreme values
(C) np(1-p)
(D) 1/λ

Correct Answer: (D)
Rate and mean waiting time are reciprocals.

Q17. Pearson correlation lies in

(A) [-1,1]
(B) association, not causation
(C) the middle ordered value
(D) λ

Correct Answer: (A)
It is a standardized covariance.

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Q18. Correlation proves

(A) $(\bar{x},\bar{y})$
(B) association, not causation
(C) the original data units
(D) n is large and p is small

Correct Answer: (B)
Confounding and reverse direction remain possible.

Q19. Regression lines intersect at

(A) rejecting a true H0
(B) n-1
(C) $(\bar{x},\bar{y})$
(D) symmetric about μ

Correct Answer: (C)
Both least-squares equations pass through the mean point.

Q20. Type I error means

(A) parameter
(B) fixed independent trials with constant p
(C) 0 and 1
(D) rejecting a true H0

Correct Answer: (D)
Its probability is α.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready only when its symbols, domain, assumptions and validity conditions are clear.

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CHAPTER 5 — DIFFERENCE EQUATIONS AND LINEAR PROGRAMMING

1. Introduction

Difference equations model change at discrete steps. Linear programming allocates limited resources using a linear objective and linear constraints. Both topics demand verification: substitute a sequence into its recurrence, and test every proposed optimum against every constraint.

2. Complete Theory

Sequences, operators and classification

A sequence y_n is a function on integer indices. The forward difference $\Delta y_n=y_{n+1}-y_n$ and shift operator E gives $Ey_n=y_{n+1}$, so $\Delta=E-1$.

Higher differences repeat the operator. A difference equation relates shifted sequence values.

Its order is the largest index gap after standard arrangement; degree is the power of the highest-order difference when polynomial in differences. Linear equations contain y terms only to first power without products.

Homogeneous equations have zero forcing; constant coefficients do not depend on n.

Hinglish Explanation

Differential equation continuous change, difference equation step-to-step change. Δ current se next ka change.

Order index gap se identify karo, sirf number of terms se nahi.

Linear recurrences and characteristic roots

For a linear homogeneous recurrence with constant coefficients, try $y_n=r^n$. This gives a characteristic polynomial.

Distinct roots r_i produce $\Sigma C_i r_i^n$. A root r repeated m times contributes $(C_0+C_{1n}+\dots+C_{m-1}n^{m-1})r^n$.

Complex conjugate roots $\rho e^{\pm i\theta}$ give real oscillatory terms $\rho^n[C\cos(n\theta)+D\sin(n\theta)]$. Initial conditions determine constants.

Substitution into the original recurrence is the decisive check.

Hinglish Explanation

ODE auxiliary equation jaisa discrete characteristic equation. Repeated root par n factors, complex root par oscillation.

Final sequence ko n and n+1 values ke through recurrence mein verify karo.

Non-homogeneous recurrences and stability

A non-homogeneous solution is complementary plus particular. Guess a particular form matching polynomial, exponential or sinusoidal forcing, modified by powers of n when it overlaps the complementary solution.

For $y_{n+1}=ay_n+b$, equilibrium $y^*=b/(1-a)$ when $a\neq1$. Deviations obey $e_{n+1}=ae_n$, so $|a|<1$ gives asymptotic stability, $|a|>1$ instability, and negative a produces alternating approach or divergence.

Applications include compound growth, inventory and iterative numerical methods.

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Hinglish Explanation

Particular guess CF se overlap kare to n multiply. Stability root magnitude se: unit circle ke andar decay, outside growth.

Negative root sign alternate karata hai.

LPP formulation and assumptions

Choose decision variables with units. Express profit, cost or time as a linear objective Z.

Translate every resource or requirement into a linear equality or inequality and add non-negativity. A feasible solution satisfies all constraints; an optimal solution is best among feasible solutions.

LPP assumes proportionality, additivity, divisibility, certainty and non-negativity. These assumptions can fail through setup costs, interactions, indivisible crews or uncertain demand, so the mathematical optimum is conditional on the model.

Hinglish Explanation

Word problem mein pehle variables and units. Then objective, constraints, $x \geq 0$.

Profit maximize aur resource limits usually ≤; demand minimum often ≥, but wording decide karega. Model assumptions practical truth nahi, approximation hain.

Graphical solution and special cases

For two variables, replace each inequality by its boundary line, test a point to select the correct half-plane and intersect all half-planes with non-negativity. The feasible region is convex.

A bounded linear objective reaches an optimum at a corner point, though an entire edge can be optimal. Evaluate Z at every feasible corner.

Parallel objective and boundary may give multiple optima; no common region means infeasible; unlimited improvement gives unbounded; a constraint that never changes the feasible region is redundant. Unbounded region does not automatically mean unbounded objective.

Hinglish Explanation

Graphical method mein shading individually nahi, common overlap feasible region. Har vertex all constraints satisfy kare.

Unbounded feasible region ke baad objective direction check karo; optimum phir bhi finite ho sakta hai.

Slack, simplex and duality basics

A ≤ resource constraint becomes equality by adding nonnegative slack. A ≥ constraint subtracts surplus and may need an artificial variable to obtain an initial basis.

In simplex, nonbasic variables are set to zero, basic variables define a corner, an entering variable improves the objective and a leaving variable is selected by the positive minimum-ratio test. Pivot row operations create a new basis.

Big-M or two-phase methods remove artificial variables. The dual assigns shadow values to primal constraints; under suitable feasibility, primal and dual optimal values agree.

Hinglish Explanation

Slack unused resource. Simplex corner se adjacent improving corner move karta hai.

Ratio test mein only positive denominator ratios. Artificial variable original resource nahi, starting basis device hai aur final solution se remove honi chahiye.

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Transportation as an LPP application

A transportation table allocates shipments from supplies to demands at minimum cost. Balanced problems have equal total supply and demand; add a zero-cost dummy row or column to balance an unbalanced problem when appropriate.

North-west corner gives an initial basic feasible solution without costs; least-cost uses cheap cells; Vogel approximation uses penalties and often starts better. An $m \times n$ nondegenerate basic solution has $m+n-1$ occupied independent cells.

Initial feasibility is not necessarily optimality.

Hinglish Explanation

NW corner cost ignore karta hai; least-cost cheapest cell; VAM penalty use karta hai. Dummy balance ke liye, but dummy allocation ka practical meaning check karo.

Initial solution ko optimum assume mat karo.

4. Formula Section

Formula Sheet
1. $\Delta y_n = y_{n+1} - y_n$
2. $E\, y_n = y_{n+1}$
3. $\Delta = E - 1$
4. homogeneous trial $y_n = r^n$
5. distinct roots $y_n = \Sigma C_i\, r_i^n$
6. repeated root r multiplicity m gives $n^k\, r^n, k=0...m-1$
7. equilibrium $y^* = b/(1-a)$ for $y_{n+1} = ay_n + b$
8. stable if $\lvert a \rvert < 1$
9. LPP $Z = c_{1x1} + c_{2x2}$
10. feasible means all constraints satisfied
11. $\le$ constraint +slack=equality
12. $\ge$ constraint -surplus=equality
13. transportation balance Σsupply=Σdemand
14. nondegenerate basic cells=$m+n-1$

5. Derivations and Solved Examples

Solve y_{n+2}-3y_{n+1}+2y_n=0. Characteristic r²-3r+2=(r-1)(r-2)=0, so $y_n=C_1+C_2\ 2^n$. If $y_0=1, y_1=3$, then $C_1+C_2=1$ and $C_1+2C_2=3$, giving $C_2=2, C_1=-1$. For LPP maximize Z=3x+2y subject to x+y≤4,x≤2,x,y≥0, corners are (0,0),(2,0),(2,2),(0,4); Z values 0,6,10,8, so optimum is (2,2).

6. Practical Applications

- Population and compound-growth models.
- Inventory and loan-balance recurrences.
- Construction resource allocation.
- Production and labour planning.
- Transportation and distribution.

7. Short Tricks

- Characteristic roots control long-term behaviour.
- Every LPP corner must satisfy every constraint.
- Parallel objective edge may mean multiple optima.
- Slack measures unused ≤ resource.

8. Concept Diagram

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LPP GRAPHICAL FLOW
variables+units -> objective -> inequalities -> half-planes
-> common feasible region -> list corners -> evaluate Z -> classify optimum
RECURRENCE FLOW
classify -> characteristic roots -> CF -> particular -> initial data -> substitute check.
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9. Comparison Table

Concept	Difference equation	LPP
Unknown	sequence y_n	decision variables
Core condition	recurrence across indices	linear constraints
Verification	substitution	feasibility and objective
Special issue	root magnitude	bounded/infeasible/multiple

10. Common Mistakes

- Using term count as recurrence order.
- Forgetting n multiplier at resonance.
- Shading the wrong inequality side.
- Evaluating non-feasible corners.
- Assuming an initial transportation solution is optimal.

11. Important DSSSB Exam Points

- Operator relations, root cases, stability, LPP formulation, graphical special cases and simplex vocabulary are frequent.

12. Chapter Revision Sheet

For recurrences, solve and substitute. For LPP, define units, graph every half-plane, verify corners, evaluate the objective and classify special cases before declaring an optimum.

13. Practice Questions with Solutions

Attempt each question first. The explanation discusses only the correct answer.

Q1. Forward difference Δy_n equals

(A) $y_{n+1}-y_n$
(B) $Ey_n=y_{n+1}$
(C) alternating convergence
(D) infeasible problem

Correct Answer: (A)
It measures one-step change.

Q2. The shift operator E gives

(A) $\Delta=E-1$
(B) $Ey_n=y_{n+1}$
(C) linear in decision variables
(D) unbounded solution

Correct Answer: (B)
E advances the index.

Q3. The operator relation is

(A) $y_n=r^n$
(B) all constraints
(C) $\Delta=E-1$
(D) redundant

Correct Answer: (C)
Subtracting identity from shift gives forward difference.

Q4. A trial for constant-coefficient homogeneous recurrence is

(A) $C_i r_i^n$
(B) decision variables are at least zero
(C) a slack variable
(D) $y_n=r^n$

Correct Answer: (D)
It converts shifts into a polynomial in r.

Q5. Distinct roots give terms of the form

(A) $C_i r_i^n$
(B) n
(C) two decision variables
(D) the positive minimum-ratio test

Correct Answer: (A)
Each characteristic root contributes a mode.

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Q6. A repeated root requires additional factors of

(A) $|a|<1$
(B) n
(C) a corner point or optimal edge
(D) total supply equals total demand

Correct Answer: (B)
Polynomial factors in n create independent solutions.

Q7. A stable first-order equilibrium requires

(A) alternating convergence
(B) infeasible problem
(C) $|a|<1$
(D) m+n-1 occupied independent cells

Correct Answer: (C)
Deviations ae_n decay.

Q8. A negative stable root produces

(A) linear in decision variables
(B) unbounded solution
(C) $y_{n+1}-y_n$
(D) alternating convergence

Correct Answer: (D)
The sign changes while magnitude decays.

Q9. An LPP objective function is

(A) linear in decision variables
(B) all constraints
(C) redundant
(D) $Ey_n=y_{n+1}$

Correct Answer: (A)
Linearity defines the model.

Q10. A feasible solution satisfies

(A) decision variables are at least zero
(B) all constraints
(C) a slack variable
(D) $\Delta=E-1$

Correct Answer: (B)
Feasibility precedes optimality.

Q11. Non-negativity restrictions mean

(A) two decision variables
(B) the positive minimum-ratio test
(C) decision variables are at least zero
(D) $y_n=r^n$

Correct Answer: (C)
Negative production or allocation is excluded.

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Q12. The graphical method directly handles

(A) a corner point or optimal edge
(B) total supply equals total demand
(C) $C_i r_i^n$
(D) two decision variables

Correct Answer: (D)
Its feasible region is drawn in a plane.

Q13. A bounded feasible LPP has an optimum at

(A) a corner point or optimal edge
(B) infeasible problem
(C) m+n-1 occupied independent cells
(D) n

Correct Answer: (A)
Linear objectives attain extrema at extreme points.

Q14. No common feasible region means

(A) unbounded solution
(B) infeasible problem
(C) $y_{n+1}-y_n$
(D) $|a|<1$

Correct Answer: (B)
No point satisfies all constraints.

Q15. Unlimited objective improvement means

(A) redundant
(B) $Ey_n=y_{n+1}$
(C) unbounded solution
(D) alternating convergence

Correct Answer: (C)
No finite optimum exists in that direction.

Q16. A constraint that does not change the feasible region is

(A) a slack variable
(B) $\Delta=E-1$
(C) linear in decision variables
(D) redundant

Correct Answer: (D)
Other constraints already imply it.

Q17. A ≤ constraint receives

(A) a slack variable
(B) the positive minimum-ratio test
(C) $y_n=r^n$
(D) all constraints

Correct Answer: (A)
Slack measures unused resource.

Q18. The simplex leaving variable uses

(A) total supply equals total demand
(B) the positive minimum-ratio test
(C) $C_i r_i^n$
(D) decision variables are at least zero

Correct Answer: (B)

It preserves feasibility.

Q19. Balanced transportation requires

(A) m+n-1 occupied independent cells
(B) n
(C) total supply equals total demand
(D) two decision variables

Correct Answer: (C)

A dummy can balance unequal totals.

Q20. A nondegenerate m×n transportation basis has

(A) $y_{n+1}-y_n$
(B) $|a|<1$
(C) a corner point or optimal edge
(D) m+n-1 occupied independent cells

Correct Answer: (D)

This is the required basic count.

Quick Revision: Re-read the formula box, cover the answers, and explain each exam point aloud. A formula is ready only when its symbols, domain, assumptions and validity conditions are clear.

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